STRENGTH OF MATERIALS-II

STUDY MATERIAL

MODULE-I

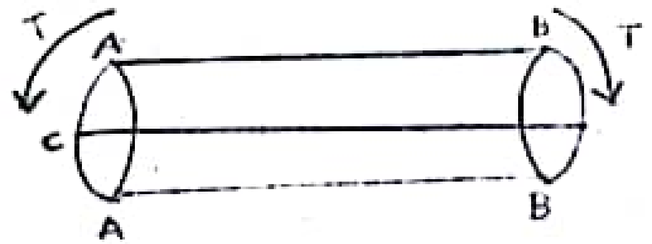
TORSION, SHAFTS & SPRINGS, BEAMS CURVED IN PLAN

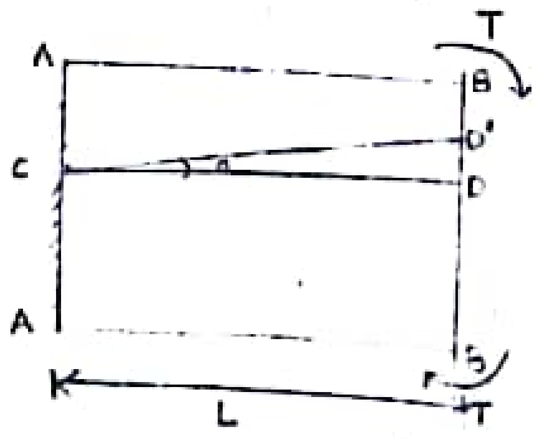
TORSION OF SHAFTS

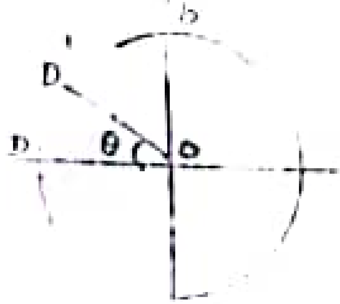
Introduction :-

A shaft is said to be in a torsion when equal and opposite torques applied at the two ends of the shaft. The torque is equal to the product of force applied and radius of the shaft. Due to the application of torque at the two ends, the shaft is subjected to twisting moment. This causes shear stress and shear strain in the material of the shaft.

Derivation to shear stress produced in a circular shaft subjected to Torsion :-







when a circular shaft is subjected to torsion shear stresses are set up in the material. To determine the magnitude of shear stress at any point of the shaft. Consider a shaft which is fixed at one end (AA) and free at another end (BB).

Let CD is anyline on the outer surface of shaft. Now, let the shaft is subjected to torque T at free end BB. As a result of this torque the shaft will rotate in clockwise direction at BC and the cross-section of the shaft is subjected to shear stress.

The point D is shifted to D' and hence the line CD is shifted to CD' and the line OD is shifted to OD', as shown in fig.

Let, T = Torque applied at end BB

τ = shear stress induced due to torque

R = Radius of shaft

L = Length of the shaft

ϕ = $\angle DCD'$, shear strain

θ = Angle of twist (radians)

C = Modulus of rigidity.

Now, shear strain at the outersurface

ϕ = Distortion of outersurface / original length

$$\phi = \frac{CD' - CD}{\cdot} = \frac{DD'}{\cdot}$$

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From Δ^{le} CDD', $\tan\phi = DD'/CD$

$$\tan\phi \simeq \phi$$

$$\phi = \frac{DD'}{CD}$$

$$\phi = \frac{DD'}{L}$$

Now, $DD' = R\theta$

Now, $\frac{DD'}{L} = \frac{R\theta}{L} = \phi$

But, C = shear stress / shear strain

$$C = \tau / \frac{R\theta}{L}$$

$$C = \tau L / R\theta$$

$$\boxed{\frac{\tau}{R} = \frac{C\theta}{L}} \text{ --- } (1)$$

Now, for a given section of shaft subjected to torque the values of C, θ, and L are constants. Hence, shear stress produced is directly proportional to the radius of shaft

$$\tau \propto R$$

$$\tau = kR$$

$$\boxed{\frac{\tau}{R} = k} \text{ --- } (2)$$

Hence, if q is shear stress induced at radius 'r' from the centre of the shaft then,

$$\boxed{\frac{q}{r} = k} \text{ --- } (3)$$

Combining (1) & (2) & (3) we get

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r}$$

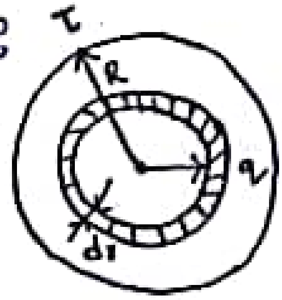
From eqn ② it is clear that shear stress is directly proportional to distance of the point from the axis of shaft. Hence, shear stress is max at the outer surface of the shaft and is min at the centre of the shaft.

Assumptions made in theory of pure Torsion:-

1. Material of the shaft is uniform throughout the length.
2. The twist along the shaft is uniform
3. The cross-section of the shaft which is plained before twist remains plain after twist
4. The shaft is uniform circular section throughout.

Derivation of maximum Torque transmitted by a circular shaft:-

Consider an elementary ring of thickness dr at a dist r from the center



Now, $\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r}$

$$\frac{\tau}{R} = \frac{q}{r}$$

$$q = \frac{\tau}{R} \times r$$

Area of elementary ring, $dA = 2\pi r dr$

Now,
w.k.t shear stress = $\frac{\text{shear force}}{\text{Area}}$

$$q = F/dA$$

$$F = q \times dA$$

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$$F = \frac{\tau}{R} \times r \times 2\pi r\, dr$$

$$F = \frac{\tau}{R} \times 2\pi r^2\, dr$$

Now, for elementary ring, Torque $dT = F \times r$

$$dT = \tau/R \times 2\pi r^2 \times r\, dr$$

$$dT = \tau/R \times 2\pi r^3\, dr$$

Now, for total shaft,

$$\text{Torque } T = \int_0^R dT$$

$$T = \int_0^R 2\pi r^3 \times \frac{\tau}{R}\, dr$$

$$T = \frac{\tau}{R}\left(\frac{2\pi r^4}{4}\right)_0^R$$

$$T = \frac{\tau}{R} \times \left(\frac{\pi r^4}{2}\right)_0^R$$

$$T = \frac{\tau}{R} \times \frac{\pi}{2} \times R^4$$

$$T = \frac{\pi}{2} \times \tau R^3$$

$$T = \frac{\pi}{2} \times \tau\,(D/2)^3$$

$$T = \frac{\pi}{2} \times \tau \times \frac{D^3}{8}$$

$$T = \frac{\pi}{16}\tau D^3$$

$$T_{max} = \frac{\pi}{16}\tau D^3$$ For a solid circular section

Maximum Torque for hollow circular shaft is

$$T = \frac{\pi}{16}\tau\left(\frac{D_o^4 - D_i^4}{D_o}\right)$$

Expression for Torque in terms of polar moment of Inertia :-

Polar moment of Inertia of a plane area is defined as MOI of the area about an axis ⊥r to the plane and passing through the CG of area. It is denoted by 'J.'

w.k.t

$$\frac{\tau}{R} = \frac{q}{r}$$

$$q = \frac{\tau}{R} \times r$$

Area of an elementary ring 'dA' = $2\pi r dr$

Now,

$$\text{shear stress} = \frac{S.F}{Area} = \frac{F}{dA}$$

$$F = \text{shear stress} \times dA$$

$$= \frac{\tau}{R} \times r \times 2\pi r \, dr$$

$$F = \frac{\tau}{R} \times 2\pi r^2 dr$$

Now,

$$dT = F \times r$$

$$= \frac{\tau}{R} \times 2\pi r^2 \times r \, dr$$

$$dT = \frac{\tau}{R} 2\pi r^3 dr$$

$$T = \int_0^R dT$$

$$T = \int_0^R \frac{\tau}{R} 2\pi r\, r^2 dr$$

$$T = \frac{\tau}{R} 2\pi r \int r^2 dr$$

$$T = \frac{\tau}{R} \int r^2 \times 2\pi r\, dr$$

$$T = \frac{\tau}{R} \int r^2 dA \qquad [\because 2\pi r\, dr = dA]$$

$\because \int r^2 dA$ is the moment of Inertia of a circular ring passing through the CG of the shaft. This is called as polar moment of Inertia [J]

$$\therefore \int r^2 dA = J$$

$$T = \frac{\tau}{R} \times J$$

$$\frac{T}{J} = \frac{\tau}{R} \quad \text{———} \quad (1)$$

But, w.k.t

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r} \quad \text{———} \quad (2)$$

Now, combining (1) & (2), we get

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r}}$$

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Note :-

1. Polar moment of Inertia for a solid circular section is $J = \frac{\pi}{32} D^4$

2. Polar moment of Inertia for a hollow circular section is $J = \frac{\pi}{32} [D_o^4 - D_i^4]$

Here, T = Torque applied on the shaft

J = Polar Moment of Inertia

Polar section Modulus :-

Polar section modulus is defined as the ratio of Polar MOI to the radius of the shaft. It is also called as Torsional section modulus. It is denoted by Z_p.

$$\therefore Z_p = \frac{J}{R}$$

For a solid circular shaft,

$$Z_p = \frac{\frac{\pi}{32} D^4}{\frac{D}{2}}$$

$$\boxed{Z_p = \frac{\pi}{16} D^3}$$

For a hollow circular shaft,

$$Z_p = \frac{\frac{\pi}{32}[D_o^4 - D_i^4]}{\frac{D_o}{2}}$$

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$$\boxed{Z_p = \frac{\pi}{16 D_o}[D_o^4 - D_i^4]}$$

Torsional Rigidity (or) stiffness of the shaft :-

It is defined as the product of Modulus of Rigidity and polar moment of Inertia

$$\boxed{T = C \times J}$$

(or)

Torsional rigidity is also defined as the Torque required to produce a twist of one radian per unit length of a shaft

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$T = \frac{C\theta J}{L}$$

$$T = \frac{C \times 1 \times J}{1}$$

$$T = C \times J$$

$$\boxed{T = CJ}$$

Note :-

$$\frac{T}{J} = \frac{\tau}{R}$$

$$T = \frac{\tau}{R} \times J$$

$$T = \tau \times Z_p$$

$$\boxed{T = Z_p \tau}$$

Power transmitted by the shaft :-

Consider a shaft which is subjected to a Torque T and rotating at a speed of 'N' rpm.

The power transmitted by the shaft is given by

$$P = \omega T$$

where, P = power transmitted by the shaft

ω = Angular speed of the shaft

$\omega = \frac{2\pi N}{60}$

T = Mean Torque

$$\therefore \boxed{P = \frac{2\pi N}{60} \times T}$$

Pblms

Formulas

1. $\frac{T}{R} = \frac{C\theta}{L} = \frac{q}{r}$

2. $T = \frac{\pi}{16} \tau D^3$

3. $T = \frac{\pi}{16 D_o} \tau [D_o^4 - D_i^4]$

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4. $\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r}$

5. $Z_p = \frac{J}{R} = \frac{\pi}{16} D^3$

6. $Z_p = \frac{\pi}{16 D_o} [D_o^4 - D_i^4]$

7. $T = CJ$

8. $T = Z_p \tau$

9. $P = \frac{2\pi N T}{60}$

10. $J = \frac{\pi}{32} [D^4]$

11. $J = \frac{\pi}{32} [D_o^4 - D_i^4]$

① The shear stress of a solid shaft is not to exceed $40 N/mm^2$, when the torque transmitted is 20,000 N-m. Determine the minimum diameter of the shaft.

Sol- Given

$\tau = 40 N/mm^2$

$T = 20{,}000 N/m$

$= 20{,}000 \times 10^3$ N-mm

$T = 20 \times 10^6$ N-mm

$D = ?$

w.k.t

$$T = \frac{\pi}{16} \tau D^3$$

$$D^3 = \frac{T \times 16}{\tau \times \pi}$$

$$D^3 = \frac{20 \times 10^6 \times 16}{\pi \times 40}$$

$$D = 136.55\,mm$$

② In a hollow circular shaft of outer dia 10cm and Inner dia 5cm, the shear stress not to exceed $40 N/mm^2$. Find the Torque which the shaft can be safely transmitted.

Sol: Given

D_o = 10 cm = 100mm.

D_i = 5cm = 50mm

τ = $40 N/mm^2$

T = ?

$$T = \frac{\pi}{16 D_o} \times \tau \left[D_o^4 - D_i^4 \right]$$

$$T = \frac{\pi}{16 \times 100} \times 40 \left[100^4 - 50^4 \right] = 7363107.78 N\text{-}mm$$

$$= 7.3\ MN\text{-}mm$$

③ A solid shaft has to transmit 75kW at 200 rpm. Taking allowable shear stress as 70 N/mm². Find suitable dia of the shaft. If max Torque transmitted at each revolution exceed the mean by 30%.

Sol: Given

$P = 75 \text{ kW} = 75\times10^3 \text{ W}$

$N = 200 \text{ rpm}$

$\tau = 70 \text{ N/mm}^2$

Now,

$T_{max} = 1.3\, T_{mean}$

w.k.t

$P = \frac{2\pi NT}{60}$

$T = \frac{P\times60}{2\pi N}$

$T = \frac{75\times10^3\times60}{2\times\pi\times200}$

$T_{mean} = 3580.98 \text{ N-mm}$

Now,

$T_{max} = 1.3\, T_{mean}$

$= 1.3\times3580.98$

$T_{max} = 4655.27 \text{ N-mm}$

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Now,

w.k.t $T = \frac{\pi}{16} \tau D^3$

$$D^3 = \frac{16T}{\pi \times \tau}$$

$$= \frac{16 \times 4655.27}{\pi \times 60}$$

$D = 6.97$ mm

4. The average torque transmitted by a shaft is 2255 N-m. The max torque is 40% more than avg torque. If the allowable shear stress in the shaft is 45 N/mm², determine the dia of the shaft.

Sol:- Given data:-

$T_{avg} = 2255$ N-m $= 2255 \times 10^3$ N-mm.

$T_{max} = 1.4\, T_{avg}$

$\tau = 45$ N/mm²

$D = ?$

w.k.t

$$T = \frac{\pi}{16} \tau D^3$$

$$D^3 = \frac{16T}{\pi \tau}$$

$T_{max} = 1.4 \times 2255 \times 10^3$

$T_{max} = 3157000$ N-mm

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$$D^3 = \frac{16 \times 8157000}{\pi \times 45}$$

$D^3 = \cancel{35993.64}$ 357299.31

$D = 70.95$ mm

5. A hollow shaft 400mm external dia and 200mm internal dia is subjected to a twisting moment of 400 kN.m. If $C = 0.8 \times 10^5$ N/mm², Find the max shear stress in the shaft. Also determine the twist in a length of 10 times that of external dia.

Sol: Given

$D_o = 400$ mm

$D_i = 200$ mm

$T = 400$ kN-m

$= 4 \times 10^8$ N-mm

$G = 0.8 \times 10^5$ N/mm²

$\tau = ?$

w.k.t

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

Now,

$$\tau = \frac{16 T D_o}{\pi [D_o^4 - D_i^4]}$$

$$\tau = \frac{16 \times 4 \times 10^8 \times 400}{\pi [400^4 - 200^4]}$$

$\tau = 33.95$ N/mm²

Now,

$$L = 10D_o$$

$$= 10 \times 400 = 4000mm$$

w.k.t

$$\frac{T}{J} = \frac{C\theta}{L} = \frac{\tau}{R}$$

where Now,

$$J = \frac{\pi}{32}[D_o^4 - D_i^4]$$

$$= \frac{\pi}{32}[400^4 - 200^4]$$

$$J = 2.35 \times 10^9 \, mm^4$$

Now,

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\theta = \frac{TL}{JC} = \frac{4 \times 10^8 \times 4000}{2.35 \times 10^9 \times 0.8 \times 10^5}$$

$$\theta = 8.51 \times 10^{-3} \text{ radians}$$

$$\theta = 0.0085 \text{ radians}$$

6) In a hollow shaft, the external dia is 100mm and internal dia 60mm. Allowable shear stress in the shaft is 50 N/mm^2, determine the angle of twist in length of 20 times of external dia of the shaft. Take $G = 8 \times 10^4$ N/mm^2.

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Sol:- Given

$D_o = 100mm$

$D_i = 60mm$

$\tau = 50N/mm^2$

$C = 8 \times 10^4 N/mm^2$

w.k.t

$$T = \frac{\pi}{16}\tau\left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

$$= \frac{\pi}{16} \times 50\left[\frac{100^4 - 60^4}{100}\right]$$

$$T = 854513\overset{3}{\cancel{1}}2.01 \, N\text{-}mm$$

Now,

$L = 20D_o$

$L = 20 \times 100 = 2000 \, mm$

w.k.t

$$\frac{T}{J} = \frac{C\theta}{L} = \frac{\tau}{R}$$

Now,

$$J = \frac{\pi}{32}\left[D_o^4 - D_i^4\right]$$

$$= \frac{\pi}{32}\left[100^4 - 60^4\right]$$

$$J = 8545132.018 \, mm^4$$

Now,

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\theta = \frac{TL}{JC} = \frac{8545132.01 \times 2000}{8545132.01 \times 8 \times 10^4} = 0.025 \text{ radians}$$

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7) A 2m long hollow shaft has outer and inner dia as 20cm and 15cm resp. If the angle of twist is not to exceed half degree in 2m length, find the max power that can be transmitted by a shaft at 200rpm and also find the max. shear stress. Take $G = 8.4 \times 10^6 N/mm^2$.

Sol:- $D_o = 20cm = 200mm$

$D_i = 15cm = 150mm$

$l = 2m = 2000mm$

$N = 200rpm$

$C = 8.4 \times 10^6 N/mm^2$

$\theta = \frac{1}{2}^\circ = \frac{1}{2} \times \frac{\pi}{180} = 0.008 rad$

$P = ?$ $\tau = ?$

w.k.t $P = \frac{2\pi NT}{60}$

But $\frac{T}{J} = \frac{C\theta}{L}$

$J = \frac{\pi}{32}[D_o^4 - D_i^4]$

$J = 107378655.2 mm^4$

Now, $T = \frac{CJ\theta}{L} = \frac{8.4 \times 10^6 \times 107378655.2 \times 0.008}{2000}$

$T = 3.6 \times 10^9 N\text{-}mm$

Now, $P = \frac{2\pi NT}{60}$

$= \frac{2\pi \times 200 \times 3.6 \times 10^9}{60}$

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$P = 7.53 \times 10^{10}$ Watts

Now, $T = \frac{\pi}{16} \times \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$

$$\tau = \frac{16 T D_o}{D_o^4 - D_i^4}$$

$$= \frac{16 \times 3.6 \times 10^9 \times 200}{200^4 - 150^4}$$

$$\tau = 3352.66 \text{ N/mm}^2$$

8) A specimen 30mm dia and 260mm in length is subjected to torsion and tensile test. It was found that tensile load of 50kN produced in elongation of 0.45mm and torque of 12.5kN-cm produced in a twist of 1.5°. Determine

i) Young's Modulus (E) ii) Rigidity modulus

iii) poisson's ratio iv) Bulk Modulus

Sol: given:

$D = 30$mm

$L = 260$mm

$P = 50$kN $= 50 \times 10^3$ N

$\delta L = 0.45$mm

$T = 12.5$ kN-cm $= 12.5 \times 10^4$ N-mm

$\theta = 1.5^\circ = 1.5 \times \frac{\pi}{180} = 0.026$ rad

i) w.k.t

$$E = \frac{\sigma}{e} = \frac{\frac{P}{A}}{\frac{\delta l}{l}} = \frac{P}{A} \times \frac{l}{\delta l}$$

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$$= \frac{150\times10^3}{\frac{\pi}{4}\times30^2}\times\frac{260}{0.45}$$

$$= 40869.41$$

$$= 4\times10^4\ N/mm^2$$

ii) w.k.t

$$\frac{T}{J}=\frac{C\theta}{L}$$

$$\therefore C=\frac{TL}{J\theta}$$

$$C=\frac{12.5\times10^4\times260}{\frac{\pi}{32}\times30^4\times0.026}$$

$$C = 15719.06\ N/mm^2$$

iii) w.k.t

$$E=2C(1+\mu)$$

$$4\times10^4 = 2\times1.5\times10^4(1+\mu)$$

$$\mu=\frac{4\times10^4}{2\times1.5\times10^4}-1$$

$$\mu=0.33$$

iv) w.k.t

$$E=3k(1-2\mu)$$

$$\frac{E}{3(1-2\mu)}=k$$

$$k=\frac{4\times10^4}{3(1-2\times0.33)}$$

$$k=3.9\times10^4\ N/mm^2$$

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9) A solid shaft of 60 dia, 2.5m long is acted upon by a torque producing a stress of $2\ kN/cm^2$ with in the shaft of radius 15mm. Find the torque and angle of twist of the shaft. Take $G = 8\times10^6 N/cm^2$.

Given:

$D = 60mm$

$l = 2.5m = 2500mm$

$q = 2kN/cm^2 = \frac{2\times10^3}{10^2} = 20\ N/mm^2$

$r = 15mm$

$C = 8\times10^6\ N/cm^2 = \frac{8\times10^6}{10^2} = 8\times10^4\ N/mm^2$

$T = ?,\ \theta = ?$

$$\frac{T}{J} = \frac{q}{r}$$

$$J = \frac{\pi}{32}D^4 = \frac{\pi}{32}\times60^4 = 1272345.02\ mm^4$$

Now, $T = \frac{qJ}{r}$

$$= \frac{20\times1272345.02}{15} = 1.696\times10^6\ N\text{-}mm$$

w.k.t $\frac{T}{J} = \frac{C\theta}{L}$

$$\theta = \frac{TL}{CJ}$$

$$\theta = \frac{1.696\times10^6\times2500}{8\times10^4\times1272345.02}$$

$$\theta = 0.04\ rad$$

n(2)
10) Find the max shear stress induced in a circular shaft of 15 cm dia when the shaft transmits 150KW power at 180rpm.

Sol:- Given

$D = 15cm = 150mm$

$P = 150KW = 150\times10^3 W$

$N = 180rpm$

Now, $P = \frac{2\pi NT}{60}$

$T = \frac{60P}{2\pi N}$

$= \frac{60\times150\times10^3}{2\times\pi\times180}$

$= 7.95\times10^3 N\text{-}m$

$= 7.95\times10^6 N\text{-}mm$

w.k.t $T = \frac{\pi}{16}\tau D^3$

$\tau = \frac{T\times16}{\pi D^3} = \frac{16\times7.95\times10^6}{\pi\times150^3}$

$= 11.99 N/mm^2$

11) A Hollow shaft is to transmit 300kW power at 80rpm. If the shear stress is not to exceed 60N/mm² and the internal dia is 0.6 times that of ext dia. Find the internal and external dia by assuming max torque as 1.4 times the mean torque.

Given :-

$P = 300kW = 300\times10^3 W = 3\times10^5 W$

$N = 80rpm$

$\tau = 60N/mm^2$

$D_i = 0.6D_o$

$T_{max} = 1.4\ T_{mean}$

w.k.t $P = \frac{2\pi NT}{60}$

$80\times10^5 = \frac{2\pi\times80\times T}{60}$

$T = \frac{3\times10^5\times60}{2\pi\times80}$

$T_{mean} = 35809.86\ N\text{-}m.$

$T_{mean} = 3.5\times10^7\ N\text{-}mm$

$T_{max} = 4.9\times10^7\ N\text{-}mm.$

Now, $T = \frac{\pi}{16}\tau\left[\frac{D_o^4 - D_i^4}{D_o}\right]$

$4.9\times10^7 = \frac{\pi}{16}\times60\left[\frac{D_o^4 - (0.6D_o)^4}{D_o}\right]$

$4.9\times10^7 = \frac{\pi}{16}\times60\left[0.8704\ D_o^3\right]$

$D_o^3 = \frac{4.9\times10^7\times16}{\pi\times60\times0.8704}$

$D_o = 168.43mm$

$D_i = 0.6\times168.43$

$D_i = 101.06mm$

12) Two shafts of the same material and of same length are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of a hollow circular section, whose internal dia is 2/3rd of the outside dia and the max shear stress developed in each shaft is same, compare the weights of the shaft.

Sol: Given $D_i = \frac{2}{3} D_o$

w.k.t $T_{solid} = \frac{\pi}{16} \tau D^3$

$$T_{Hollow} = \frac{\pi}{16} \times \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

$$= \frac{\pi}{16} \times \tau \left[\frac{D_o^4 - \left(\frac{2}{3} D_o\right)^4}{D_o} \right]$$

$$= \frac{\pi}{16} \times \tau \left[\frac{D_o^4 - \frac{16}{81} D_o^4}{D_o} \right]$$

$$T_{Hollow} = \frac{\pi}{16} \times \tau \left[\frac{65}{81} D_o^3 \right]$$

But,

$$T_{solid} = T_{Hollow}$$

$$\frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times \tau \left[\frac{65}{81} D_o^3 \right]$$

$$D^3 = \frac{65}{81} D_o^3$$

$$D = \left(\frac{65}{81}\right)^{1/3} \left(D_o^3\right)^{1/3}$$

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$D = 0.929 D_o$

w.k.t, wt. density $= \frac{wt}{volume}$

$W = wt \cdot density \times vol$

$W_{solid} = \rho g \times \frac{\pi}{4} D^2 \times L$

$W_{hollow} = \rho g \times \frac{\pi}{4}\left[D_o^2 - D_i^2\right] \times L$

$= \rho g \times \frac{\pi}{4}\left[D_o^2 - \left(\frac{2}{3}D_o\right)^2\right] \times L$

$= \rho g \times \frac{\pi}{4}\left[D_o^2 - \frac{4}{9}D_o^2\right] \times L$

$= \rho g \times \frac{\pi}{4}\left[\frac{5}{9}D_o^2\right] \times L$

Now, $\frac{W_{solid}}{W_{Hollow}} = \frac{\rho g \times \frac{\pi}{4} D^2 \times L}{\rho g \times \frac{\pi}{4}\left[\frac{5}{9}D_o^2\right] \times L}$

$= \frac{D^2}{\frac{5}{9}D_o^2}$

$= \frac{(0.929 D_o)^2}{\frac{5}{9}D_o^2}$

$= \frac{0.929^2}{\frac{5}{9}}$

$= \frac{1.55}{1}$

$\frac{W_{solid}}{W_{Hollow}} = \frac{1.55}{1}$

$W_{solid} = 1.55\, W_{Hollow}$

13) A solid cylindrical shaft is to transmit 300kW power at 100rpm. If the shear stress is not exceed $80 N/mm^2$. Find its diameter

ii) What % saving in wt would be obtained if this shaft is replaced by a hollow one whose internal dia = 0.6 times external dia, the length, material and max shear stress being the same.

Given $P = 300kN = 3 \times 10^5 W$

$N = 100 rpm$

$\tau = 80 N/mm^2$

$D = ?$

Now, $P = \frac{2\pi NT}{60}$

$T = \frac{60 \times P}{2\pi N}$

$= \frac{60 \times 3 \times 10^5}{2\pi \times 100}$

$T = 28647.8 \text{ N-m}$

$T = 28467.8 \times 10^3 \text{ N-mm}$

Now, $T_{solid} = \frac{\pi}{16} \tau D^3$

$D^3 = \frac{16T}{\pi \tau}$

$D^3 = \frac{16 \times 28647.8 \times 10^3}{\pi \times 80}$

$D = 122mm$

ii) w.k.t

$$T_{hollow} = \frac{\pi}{16} \cdot \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

But $T_{solid} = T_{hollow}$ [∴ Given]

$$T_{hollow} = \frac{\pi}{16} \tau \left[\frac{D_o^4 - (0.6D_o)^4}{D_o} \right]$$

$$= \frac{\pi}{16} \tau \left[0.8704 D_o^3 \right]$$

Now, $T_{solid} = T_{hollow}$

$$28647.8 \times 10^3 = \frac{\pi}{16} \times 80 \left[0.8704 D_o^3 \right]$$

$$D_o^3 = \frac{28647.8 \times 10^3 \times 16}{\pi \times 80 \times 0.8704}$$

$$D_o = 127.96 mm$$

$$D_o = 128 mm$$

Now, $D_i = 0.6 D_o$

$$= 0.6 \times 128$$

$$= 76.8 mm$$

$$D_i = 77 mm$$

Now,

$$\% \text{saving in weight} = \frac{W_s - W_h}{W_s} \times 100$$

$$= \frac{\rho g \times \frac{\pi}{4} \times D^2 \times L - \rho g \times \frac{\pi}{4} \times [D_o^2 - D_i^2] \times L}{\rho g \times \frac{\pi}{4} \times D^2 \times L}$$

$$= \frac{D^2 - [D_o^2 - D_i^2]}{D^2} \times 100$$

$$= \frac{122^2 - [128^2 - 77^2]}{122^2} \times 100 = 29.75\%$$

14) A Hollow shaft having an inside diameter 60% of its outer diameter is to replace a solid shaft transmitting the same power at the same speed. Calculate the % saving in material, If the material is to be used is also the same.

Sol:- Given data:-

$D_i = 0.6D_o$

w.k.t

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60P}{2\pi N}$$

Now,

$$T_{max}\ solid = \frac{\pi}{16}\tau D^3$$

$$T_{max}\ Hollow = \frac{\pi}{16D_o}\tau\left[D_o^4 - D_i^4\right]$$

Now,

$$T_{max}\ hollow = \frac{\pi}{16D_o}\tau\left[D_o^4 - (0.6D_o)^4\right]$$

$$= \frac{\pi}{16D_o}\tau\left[0.8704D_o^4\right]$$

$$= \frac{\pi}{16}\tau\ 0.8704D_o^3$$

But;

$$T_{max}\ solid = T_{max}\ (Hollow)$$

$$\frac{\pi}{16}\tau D^3 = \frac{\pi}{16}\tau\ 0.8704\ D_o^3$$

$$D = (0.8704)^{1/3}(D_o^3)^{1/3}$$

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$D = 0.954 D_0$

Now,

Area of solid shaft, $A_s = \frac{\pi D^2}{4}$

Area of Hollow shaft, $A_h = \frac{\pi (D_0^2 - D_i^2)}{4}$

$$A_h = \frac{\pi}{4}\left[D_0^2 - (0.6 D_0)^2\right]$$

$$= \frac{\pi}{4}\left[0.64 D_0^2\right]$$

Now,

$$\% \text{ saving of material} = \frac{A_s - A_h}{A_s} \times 100$$

$$= \frac{\frac{\pi}{4} D^2 - \frac{\pi}{4} 0.64 D_0^2}{\frac{\pi}{4} D^2} \times 100$$

$$= \frac{\frac{\pi}{4}(0.954 D_0)^2 - \frac{\pi}{4} 0.64 D_0^2}{\frac{\pi}{4} \times (0.954 D_0)^2} \times 100$$

$$= \frac{(0.954 D_0)^2 - 0.64 D_0^2}{(0.954 D_0)^2} \times 100$$

$$= \frac{0.954^2 - 0.64^2}{0.954^2} \times 100$$

$$= 29.66\%$$

15) Determine the dia of a ~~Hollow~~ solid shaft which will transmits 300kW at 250rpm. The max shear stress should not exceed 30N/mm^2 and twist should not be more than 1° in a shaft length of 2m.

Take $C = 1 \times 10^5$ N/mm^2.

Sol:-

Given data:-

$P = 300 KW = 300 \times 10^3 W$

$N = 250 rpm$

$\tau = 30 N/mm^2$

$\theta = 1^\circ = 1 \times \frac{\pi}{180} = 0.0174 radians$

$C = 1 \times 10^5 N/mm^2$

$L = 2m = 2000mm$

$D = ?$

w.k.t

$P = \frac{2\pi NT}{60}$

$T = \frac{60P}{2\pi N}$

$T = \frac{60 \times 300 \times 10^3}{2 \times \pi \times 250}$

$T = 11459.15 N\text{-}m$

$T = 11459.15 \times 10^3 N\text{-}mm$

Now,

w.k.t 1) $T = \frac{\pi}{16} \tau D^3$

$D^3 = \frac{16T}{\pi \tau}$

$D^3 = \frac{16 \times 11459.15 \times 10^3}{\pi \times 30}$

$D = 124.83 mm$

ii) $\frac{T}{J} = \frac{C\theta}{L}$

$J = \frac{\pi}{32} D^4$

$$\frac{11459.15 \times 10^3}{\frac{\pi}{32} D^4} = \frac{1 \times 10^5 \times 0.0174}{2000}$$

$$D^4 = \frac{11459.15 \times 10^3 \times 2000}{\frac{\pi}{32} \times 1 \times 10^5 \times 0.0174}$$

$D = 107.62mm$

$\therefore$ Diameter of shaft = Max value of (i), (ii)

$D = 124.83mm$

16) A Hollow shaft having an internal dia 40% of its external diameter, transmits 562.5 kW power at 100rpm. Determine the external dia of the shaft. If the shear stress is not to exceed $60N/mm^2$ and the twist in a length of 2.5m should not exceed 1.8°. Assume Max Torque = 1.25 times Mean Torque and Modulus of Rigidity, $C = 9 \times 10^4 N/mm^2$.

Sol:- Given :-

$D_i = 0.4 D_o$

$P = 562.5 kW = 562.5 \times 10^3 W$

$N = 100rpm$

$\tau = 60N/mm^2$

$L = 2.5m = 2500mm$

$\theta = 1.3^\circ = 1.3 \times \pi/180 = 0.0226$

$C = 9 \times 10^4 N/mm^2$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60P}{2\pi N}$$

$$T = \frac{60 \times 562.5 \times 10^3}{2 \times \pi \times 100}$$

$$T = 53714.79 \text{ N-m} = 53714.79 \times 10^3 \text{ N-mm}$$

$$T_{mean} = 53714.79 \times 10^3 \text{ N-mm}$$

Given

$$T_{max} = 1.25\ T_{mean}$$

$$T_{max} = 1.25 \times 53714.79 \times 10^3 = 67143487.5 \text{ N-mm}$$

Now,

$$T = \frac{\pi}{16 D_o} \times \tau \left[D_o^4 - D_i^4\right]$$

$$T = \frac{\pi}{16 D_o} \times 60 \left[D_o^4 - (0.4 D_o)^4\right]$$

$$T = \frac{\pi}{16 D_o} \times 60 \left[0.9744 D_o^4\right]$$

$$T = \frac{\pi \times 60 \times 0.9744 D_o^3}{16}$$

$$D_o^3 = \frac{16 \times T}{\pi \times 60 \times 0.9744}$$

$$D_o^3 = \frac{16 \times 67143487.5}{\pi \times 60 \times 0.9744}$$

$$D_o = 180.17 \text{ mm}$$

$$D_i = 72.068$$

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ii) $\frac{T}{J} = \frac{C\theta}{L}$

$J = \frac{\pi}{32}[D_o^4 - D_i^4]$

$$\frac{67143487.5}{\frac{\pi}{32}[D_o^4 - 0.4D_o^4]} = \frac{9\times10^4 \times 0.0226}{2500}$$

$$\frac{67143487.5}{\frac{\pi}{32}\times 0.9744 D_o^4} = \frac{9\times10^4\times0.0226}{2500}$$

$$D_o^4 = \frac{67143487.5\times2500}{\frac{\pi}{32}\times0.9744\times9\times10^4\times0.0226}$$

$D_o = 171.38$ mm $D_i = 85.69$mm

External dia of shaft = Max value of i), ii)

$D_o = 180.17$mm

6, 8/7

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Combined Bending and Torsion :

When a shaft is transmitting torque or power, it is subjected to shear stress. At the same time, the shaft is also subjected to BM due to inertia (or) gravity loads. Due to the Bending moment, the bending stresses are also set up in the shaft. For design purpose, it is necessary to find the Principle stresses, Max shear stress and strain energy. The principle stresses and Max shear stress, when a shaft is subjected to bending and Torsion are obtained as follows:

w.k.t

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Now,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = \frac{M \times \frac{D}{2}}{\frac{\pi}{64} D^4}$$

$$\sigma_b = \frac{MD}{2} \times \frac{64}{\pi D^4}$$

$$\sigma_b = \frac{32M}{\pi D^3}$$

Now,

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{TR}{J}$$

$$\tau = \frac{T \times \frac{D}{2}}{\frac{\pi}{32} D^4}$$

$$\tau = \frac{TD}{2} \times \frac{32}{\pi D^4}$$

$$\tau = \frac{16T}{\pi D^3}$$

Now.

$$\sigma_{major} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{\frac{32M}{\pi D^3}}{2} + \sqrt{\left(\frac{\frac{32M}{\pi D^3}}{2}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

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$$= \frac{16M}{\pi D^3} + \sqrt{\left(\frac{16M}{\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$\therefore \sigma_{major} = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

now,

$$\therefore \sigma_{minor} = \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right]$$

now,

$$\tau_{max} = \frac{\sigma_{major} - \sigma_{minor}}{2}$$

$$= \frac{\frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right] - \frac{16}{\pi D^3}\left[M - \sqrt{M^2 + T^2}\right]}{2}$$

$$= \frac{16}{\pi D^3}\left[\frac{M + \sqrt{M^2 + T^2} - M + \sqrt{M^2 + T^2}}{2}\right]$$

$$= \frac{16}{\pi D^3}\left[\frac{2\sqrt{M^2 + T^2}}{2}\right]$$

$$\tau_{max} = \frac{16}{\pi D^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

$$= \frac{2 \times \frac{16T}{\pi D^3}}{\frac{32M}{\pi D^3}}$$

$$\tan 2\theta = \frac{T}{M}$$

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For a hollow shaft,

$$\sigma_{major} = \frac{16 D_o}{\pi [D_o^4 - D_i^4]} \left[M + \sqrt{M^2 + T^2} \right]$$

$$\sigma_{minor} = \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left[M - \sqrt{M^2 + T^2} \right]$$

$$\tau_{max} = \frac{16 D_o}{\pi (D_o^4 - D_i^4)} \left[\sqrt{M^2 + T^2} \right]$$

① A solid shaft of diameter 80mm is subjected to a twisting moment of 8MN-mm and BM of 5MN-mm at a point. Determine i) Principal stresses ii) Position of plane on which they act.

Sol:- Given:-

$D = 80mm$

$T = 8 MN\text{-}mm = 8 \times 10^6 N\text{-}mm$

$M = 5 MN\text{-}mm = 5 \times 10^6 N\text{-}mm$

i) w.k.t

$$\sigma_{major} = \frac{16}{\pi D^3} \left[M + \sqrt{M^2 + T^2} \right]$$

$$= \frac{16}{\pi \times 80^3} \left[5 \times 10^6 + \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2} \right]$$

$$= 143.57 N/mm^2$$

$$\sigma_{minor} = \frac{16}{\pi D^3} \left[M - \sqrt{M^2 + T^2} \right]$$

$$= \frac{16}{\pi \times 80^3} \left[5 \times 10^6 - \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2} \right]$$

$= -44.13\ N/mm^2$

$\sigma_{minor} = 44.13\ N/mm^2$ (C)

ii) w.k.t

$$Tan 2\theta = \frac{T}{M}$$

$$2\theta = \tan^{-1}\left(\frac{8\times10^6}{5\times10^6}\right)$$

$$\theta = 28.99^\circ \simeq 28^\circ 59'$$

③ The Max allowable shear stress in a hollow shaft of external diameter equal to twice the internal dia is $80 N/mm^2$. Determine the diameter of the shaft, If it is subjected to a torque of 4×10^6 N-mm and BM of 3×10^6 N/mm² N-mm.

Sol:- Given :-

$D_o = 2D_i$

$D_i = \frac{D_o}{2}$

$\tau_{max} = 80 N/mm^2$

$T = 4\times10^6$ N-mm

$M = 3\times10^6$ N-mm

w.k.t

$$\tau_{max} = \frac{16 D_o}{\pi[D_o^4 - D_i^4]} \left[\sqrt{M^2+T^2}\right]$$

$$80 = \frac{16 D_o}{\pi\left[D_o^4 - \left(\frac{D_o}{2}\right)^4\right]} \left[\sqrt{(3\times10^6)^2 + (4\times10^6)^2}\right]$$

$$80 = \frac{16 D_o}{\pi\left[D_o^4 - \frac{D_o^4}{16}\right]} \times 10^6 \sqrt{9+16}$$

$$80 = \frac{16 D_o}{\pi\left[\frac{15 D_o^4}{16}\right]} \times 5\times10^6$$

$$80 = \frac{16 D_o}{\frac{\pi \cdot 15 \times D_o^4}{16}} \times 5\times10^6$$

$$80 = \frac{16\times16}{15\pi D_o^3} \times 5\times10^6$$

$$D_o^3 = \frac{16\times16\times5\times10^6}{15\pi\times80}$$

$$D_o = 69.76\,mm$$

$$D_i = \frac{D_o}{2} = \frac{69.76}{2} = 34.88\,mm$$

II part

Springs

Definition:-

springs are the elastic members which compress under the load & regain there original shape when the load is removed.

Springs of capable of storing must strain energy without being permanently deformed. This stored energy is given of whenever required springs are used to observe

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shock & extensively applicable in automobiles railway carriages etc.

Spring material :-

springs are generally made with steel for special application. They can also made with phosphorous bronze metal & such springs will have high fatique strength, corrosion resistance & heat resistance.

Types of springs :-

They are mainly classified into 2 types :-

i) Bending spring (Laminated or leaf spring)

ii) Torsional spring (Helical spring)

Laminated springs are again classified into 2 types :-

i) semi elliptical leaf spring

ii) Quarter elliptical leaf spring

Helical springs are again classified into 2 types :-

i) closed coil helical spring

ii) open coil helical spring

Functions of spring :-

i) To control vibration & observed shocks in automobiles

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ii) To provide desired motion to machine member such as clutches, safety valves, governs.

Definitions of some terms in connection of springs :-

1) Stiffness of spring :

The force required to deflect a spring through unit deflection (or)

the force or load per unit deflection is called stiffness of spring.

$$k = \frac{P}{\delta}$$

where P = applied load

δ = deflection

2) Spring index (or) spring constant (C) :-

It is mainly applicable in helical spring.

It is defined as Mean coil dia (D) to the dia of spring wire (d)

$$C = \frac{D}{d} \text{ (or) } \frac{R}{r}$$

3) Helix angle (α) :-

It is the angle b/w spring wire & horizontal axis or line $\perp^r$ to the axis of spring.

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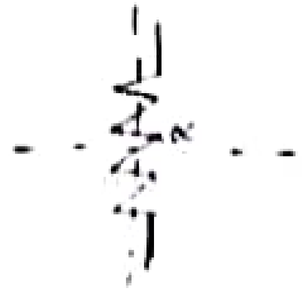


Solid length :-

It is the length of the spring measured along the axis when the spring is completely compress. i.e, coils are touching each other

Solid length = $n \times d$

where n = no. of coil

d = dia of wire

Free length :

It is the length of the spring measured along the axis when the spring is unloaded

$$\text{Free length} = nd + \delta_{max} + 0.15 \cdot \delta_{max} .$$

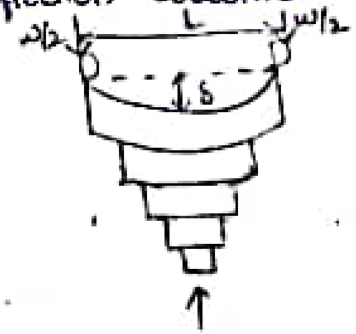
Pitch :-

The actual distance between the adjacent coil in compressive state is called pitch.

$$\text{pitch} = \frac{\text{free length}}{n-1}$$

Laminated (or) Leaf springs :-

Laminated springs are used to absorb shock loads or sudden loads in railway wagons, coaches, load carrying vehicles such as Lorries, buses, cars etc.. It consists of no. of. parallel strips of metal having different lengths, same width and same thickness which are placed one over the other. The initial position of spring is shown in the fig which is having central deflection(δ). When the spring is loaded to desired load(W) all the plates become flat, the central deflection becomes zero



Expression for Max Bending stress developed in a semi - Elliptical spring:-

The load W acting at the centre of the lower most plate will be shared equally on two ends of the plate as shown in the above figure.

Let $b \rightarrow$ width of each plate

$n \rightarrow$ no. of. plates

$L \rightarrow$ Span of the spring (Length)

$\sigma \rightarrow$ Max Bending Stress developed in the spring.

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$t \rightarrow$ thickness of the spring

$W \rightarrow$ point load acting at centre of the spring

$\delta \rightarrow$ Original deflection at the top of spring.

w.k.t

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma I}{y}$$

But,

$$I = \frac{bt^3}{12}, \quad y = t/2$$

$$M = \frac{\sigma \times \frac{bt^3}{12}}{t/2}$$

$$M = \frac{\sigma bt^3}{12} \times \frac{2}{t}$$

$$M = \frac{\sigma bt^2}{6}$$

For n plates,

$$M = \frac{\sigma n bt^2}{6} \rightarrow (1)$$

Now,

Resisting moment, $M = \frac{W}{2} \times \frac{L}{2}$

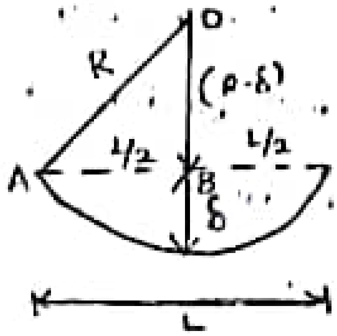
$$M = \frac{WL}{4} - (2)$$

Now, (1) = (2)

$$\frac{WL}{4} = \frac{\sigma n bt^2}{6}$$

$$\boxed{\sigma_{max} = \frac{3WL}{2nbt^2}}$$

Expression for Max deflection



Consider Δ^{le} ADB

$$R^2 = (L/2)^2 + (R-\delta)^2$$

$$R^2 = \frac{L^2}{4} + R^2 + \delta^2 - 2R\delta$$

$$2R\delta = \frac{L^2}{4} + \delta^2$$

Neglecting δ^2

$$2R\delta = L^2/4$$

$$\delta = L^2/8R$$

w.k.t

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$R = \frac{Ey}{\sigma}$$

$$R = \frac{E\left(\frac{t}{2}\right)}{\sigma}$$

$$R = \frac{Et}{2\sigma}$$

$$\therefore \delta = \frac{L^2}{8R}$$

$$\delta = \frac{L^2}{8\left(\frac{Et}{2\sigma}\right)} = \frac{2L^2\sigma}{8Et}$$

$$\delta = \frac{\sigma L^2}{4Et}$$

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A(W)

(7) A leaf spring carries a central load of 8000N. The leaf spring is to be made of 10 steel plates of 5 cm wide and 6mm thick. If the Bending stress is limited to 150 N/mm², Determine i) Length of the spring

ii) Deflection at the centre of spring

Sol: Given data:

$W = 8000N$

$n = 10$

$b = 5cm = 50mm$

$t = 6mm$

$\sigma = 150 N/mm^2$

$L = ?$

$\delta = ?$

w.k.t

$$\sigma = \frac{3WL}{2nbt^2}$$

$$L = \frac{2\sigma nbt^2}{3W}$$

$$L = \frac{2\times150\times10\times50\times6^2}{3\times8000}$$

$L = 600mm.$

ii) w.k.t

$$\delta = \frac{\sigma L^2}{4Et}$$

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$$\delta = \frac{150 \times 600^2}{4 \times 2.1 \times 10^5 \times 6}$$

$$\delta = 10.71 \text{ mm}$$

Q) A Laminated spring 1m long is made up of plates each 50mm wide and 1cm thick. If σ is limited to 100 N/mm^2, How many plates would be required to enable the spring to carry a central point load of 2kN, If $E = 2.1 \times 10^5 \text{ N/mm}^2$, what is the deflection under the load.

Sol:- Given data:-

$W = 2kN = 2 \times 10^3 N$, $L = 1m = 1000mm$

$b = 50mm$

$t = 1cm = 10 mm$

$\sigma = 100 N/mm^2$

$E = 2.1 \times 10^5 N/mm^2$

w.k.t

i) $\sigma = \frac{3WL}{2nbt^2}$

$n = \frac{3WL}{2\sigma bt^2}$

$n = \frac{3 \times 2 \times 10^3 \times 1000}{2 \times 100 \times 50 \times 10^2}$

$\boxed{n = 6}$

ii) w.k.t

$\delta = \frac{\sigma L^2}{4Et}$

$\delta = \frac{100 \times 1000^2}{4 \times 2.1 \times 10^5 \times 10}$

$\boxed{\delta = 11.90 \text{ mm}}$

Expression for max bending stress developed in Quarter elliptical leaf spring:-

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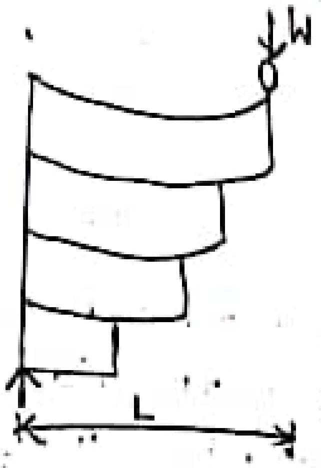
w.k.t

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$



$$\therefore \frac{M}{I} = \frac{\sigma}{y}$$

$$\therefore M = \frac{\sigma I}{y}$$

But, $\because I = \frac{bt^3}{12}$, $y = t/2$

$$M = \frac{\sigma \frac{bt^3}{12}}{t/2}$$

$$M = \frac{\sigma bt^2}{6}$$

For n plates,

$$M = \frac{\sigma nbt^2}{6} \quad \text{—} ①$$

But,

Resisting moment,

$$M = WL \quad \text{—} ②$$

Now,

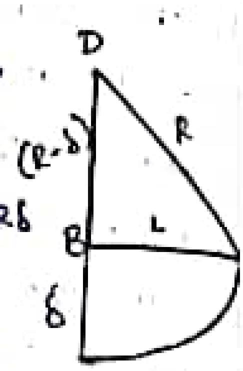
① = ②

$$WL = \frac{\sigma nbt^2}{6}$$

$$\boxed{\sigma_{max} = \frac{6WL}{nbt^2}}$$

Expression for max deflect

Consider Δ^{le} AOB,



$$R^2 = L^2 + (R-\delta)^2$$

$$R^2 = L^2 + R^2 + \delta^2 - 2R\delta$$

$$2R\delta = L^2 + \delta^2$$

Neglecting δ^2

$$2R\delta = L^2$$

$$\delta = L^2/2R$$

w.k.t

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$R = \frac{Ey}{\sigma}$$

$$R = \frac{E(t/2)}{\sigma}$$

$$R = \frac{Et}{2\sigma}$$

$$\therefore \delta = \frac{L^2}{2R}$$

$$\delta = \frac{L^2}{2\left(\frac{Et}{2\sigma}\right)}$$

$$\delta = \frac{2L^2\sigma}{2Et}$$

$$\boxed{\delta_{max} = \frac{\sigma L^2}{Et}}$$

1) A Quarter Elliptical leaf spring is 15 m long. The plate have the width 12 times of the thickness of the plate. Initially the deflection was 40cm and the load is 12.5 KN. The Bending stress in leaf spring is 150 N/mm². If $E = 2\times10^5$ N/mm², Determine the size of the plate and no. of plates.

sol:- **Given data :-**

$L = 15m = 15000mm$

$b = 12t$

$\delta = 40cm = 400mm$

$W = 12.5KN = 12.5\times10^3 N$

$E = 2\times10^5 N/mm^2$

$\sigma = 150 N/mm^2$

i) w.k.t

$$\delta_{max} = \frac{\sigma L^2}{Et}$$

$$t = \frac{\sigma L^2}{E\delta}$$

$$t = \frac{150\times15000^2}{2\times10^5\times400}$$

$t = \cancel{0.042}$ mm 421.8

Now,

$b = 12t = 12\times\cancel{0.042}$ 421.8

$= \cancel{0.5 mm}$ 5061.6mm

∴ The size of plate.

$= \cancel{0.5\times0.042}$mm

421.8 × 5061.6

ii) w.k.t

$$\sigma_{max} = \frac{6WL}{nbt^2}$$

$$n = \frac{6\times12.5\times10^3\times15000}{150\times\cancel{0.5}\times\cancel{0.042}^2}$$

5061.6 × 421.8

$n = 3.51$

$n = 4$

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HELICAL SPRING :

Helical spring is classified into 2 types

i) closed - coil helical spring

ii) open - coil helical spring

closed - coil helical spring :-

These are the springs in which helix angle is very small or equal to zero. In other words, pitch b/w the adjacent coils is zero.

In closed - coil helical spring as the helix angles is small, Hence Bending effect on spring is ignored. We assume that these springs withstand by purely torsional stresses.

Expression for Max shear stress in a closed coil helical spring :

Let $d \rightarrow$ dia of spring wire

$p \rightarrow$ pitch

$n \rightarrow$ No. of turns (or) coils

$R \rightarrow$ Mean radius of the spring

$W \rightarrow$ axial load

$C \rightarrow$ Modulus of Rigidity.

$c \rightarrow$ spring Index (or) constant

$\tau \rightarrow$ Max shear stress

$\delta \rightarrow$ Deflection of the spring

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Now, The twisting moment of spring is given as $T = W \times R$

w.k.t

$$T = \frac{\pi}{16} \tau d^3$$

$$\tau = \frac{16T}{\pi d^3}$$

Now,

$$\tau = \frac{16 \times WR}{\pi d^3}$$

$$\tau_{max} = \frac{16WR}{\pi d^3}$$

Expression for deflection of spring:

Length of one coil = $2\pi r$

The total length of coil (or) wire $= n \times 2\pi r$

As, every section of the wire is subjected to torsion so, the strain energy stored by the spring due to torsion is given as

$$U = \frac{\tau^2}{4C} \times \text{volume}$$

$$U = \frac{\tau^2}{4C} \times (\text{Area} \times \text{Length})$$

$$= \left[\frac{16WR}{\pi d^3}\right]^2 \times \frac{1}{4C} \times \left[\frac{\pi}{4} d^2 \times n \times 2\pi R\right]$$

$$= \frac{16 \times 16 \times W^2 R^2}{\pi^2 d^6} \times \frac{1}{4C} \times \frac{\pi d^2}{4} \times n \times 2\pi R$$

$$= \frac{32 W^2 R^3 n}{d^4 C}$$

$$U = \frac{32 W^2 R^3 n}{C d^4}$$

Now,

work done = Avg. load × deflection

$$\frac{W}{2} \times \delta$$

Now,

$$\frac{W}{2} \times \delta = U$$

$$\frac{W\delta}{2} = \frac{32W^2R^3n}{Cd^4}$$

$$\delta = \frac{64W^2R^3n}{Cd^4W}$$

$$\boxed{\delta_{max} = \frac{64WR^3n}{Cd^4}}$$

Now, stiffness $k = \frac{W}{\delta}$

$$k = \frac{W}{\frac{64WR^3n}{Cd^4}}$$

$$k = \frac{Cd^4W}{64WR^3n}$$

$$\boxed{k = \frac{Cd^4}{64R^3n}}$$

19) A closed coil helical spring is to carry a load of 500N. It's mean coil dia to be 10 times that of wire diameter. Calculate these diameters if the max shear stress is 80N/mm².

Sol:- Given data :-

$W = 500N$

$D = 10d$

$\tau_{max} = 80N/mm^2$

Now,

w.k.t $\tau = \frac{16WR}{\pi d^3}$

$$\tau = \frac{16\times500\times(\frac{10d}{2})}{\pi d^3}$$

$$80 = \frac{16\times500\times5d}{\pi d^3}$$

$$d^2 = \frac{16\times500\times5}{\pi\times80}$$

$$d = 12.61mm$$

Now,

Mean coil dia, $D = 10d$

$$D = 10\times12.61$$

$$D = 126.1mm$$

A(3) ~~15~~ A closed coil helical spring of round steel wire 10mm in dia having 10 complete turns with a mean dia of 12cm is subjected to an axial load of 200N. Determine i) Deflection of the spring ii) Max shear stress in wire iii) stiffness of the spring

Take $C = 8\times10^4\ N/mm^2$.

Sol:- Given data :-

$d = 10mm$

$D = 12cm = 120mm$

$W = 200N$

$C = 8\times10^4\ N/mm^2$

$n = 10$

i) w.k.t

$$\delta_{max} = \frac{64WR^3n}{Cd^4} = \frac{64W(D/2)^3n}{Cd^4} = \frac{64\times200\times(\frac{120}{2})^3\times10}{8\times10^4\times10^4}$$

$$= 34.56mm$$

ii) $\tau = \frac{16WR}{\pi d^3}$

$\tau = \frac{16W \times D/2}{\pi d^3}$

$\tau = \frac{16 \times 200 \times 120/2}{\pi \times 10^3}$

$\tau = 61.11\ N/mm^2$

iii) w.k.t

$k = \frac{Cd^4}{64R^3n}$

$k = \frac{8 \times 10^4 \times 10^4}{64 \times \left(\frac{120}{2}\right)^3 \times 10}$

$\therefore k = 5.78\ N/mm$

Q) A closed coil helical spring of mean dia 20cm is made of 3 cm dia of rod and has 16 turns. A weight of 3KN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring may be compressed by 18cm. Take $C = 8 \times 10^4\ N/mm^2$.

Sol:- Given data :-

$D = 20cm = 200mm$; $R = 100mm$

$d = 3cm = 30mm$

$n = 16$ turns

$C = 8 \times 10^4\ N/mm^2$

weight falling, $W = 3KN = 3000N$

$\delta = 18cm = 180mm$

W.L.6

Workdone = Energy stored

Here,

Energy stored is potential energy

But, P.E = mgh

$= W(h+\delta)$

$P.E = 8000(h+180)$

Now,

Work done = Avg. load × deflection

$= W/2 \times \delta$

But, W can be obtained from

$$\delta_{max} = \frac{64WR^3n}{cd^4}$$

$$180 = \frac{64 \times W \times 100^3 \times 16}{8 \times 10^4 \times 30^4}$$

$$\frac{1}{W} = \frac{64 \times 100^3 \times 16}{180 \times 8 \times 10^4 \times 30^4}$$

$$W = \frac{180 \times 8 \times 10^4 \times 30^4}{64 \times 100^3 \times 16}$$

$W = 11390.625N$

Now, work done $= \frac{W}{2} \times \delta$

$= \frac{11390.625}{2} \times 180$

$W = 1025156.25$

Now,

Workdone = P.E

$1025156.25 = 8000(h+180)$

$$h = \frac{1025156.25}{8000} - 180 = 161.71mm$$

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9) The stiffness of a closed coil helical spring is 1.5 N/mm of compression under a max load of 60N. The max shear stress produced in a wire of spring is 125N/mm². The solid length of a spring is given as 5cm. Find the following terms
i) diameter of the wire ii) Mean dia of spring
iii) No. of coils. Take $C = 4.5\times10^4$ N/mm².

Sol:- Given data :-

$k = 1.5$ N/mm

$W = 60$ N

$\tau = 125$ N/mm²

solid length $n\times d$ = 5cm = 50mm — ①

w.k.t

solid length = nd

$50 = nd$

$n = \frac{50}{d}$ — ①

Now,

$$k = \frac{Cd^4}{64R^3n}$$

$$1.5 = \frac{4.5\times10^4 \times d^4}{64R^3\times\frac{50}{d}n}$$

$$1.5 = \frac{4.5\times10^4 d^4}{64\times R^3\times n}$$

$$d^4 = \frac{64\times1.5\times R^3 n}{4.5\times10^4}$$

$$d^4 = 0.0021R^3n \text{ — ②}$$

Now,

w.k.t

$$\tau = \frac{16WR}{\pi d^3}$$

$$R = \frac{\pi d^3\tau}{16W}$$

$$R = \frac{\pi d^3\times125}{16\times60}$$

$$R = 0.409d^3 \text{ — ③}$$

Now put ③ in ②

$$d^4 = 0.0021(0.409d^3)^3 n$$

$$d^4 = 0.0021\times0.068\times d^9\times\frac{50}{d}$$

$$d^4 = \frac{1}{0.0021\times0.068\times50}$$

$d = 3.45$mm

Now,

No. of turns, $n = \frac{50}{d} = \frac{50}{3.45}$

$n = 14.53 \cong 15$

Now,

$R = 0.409d^3$

$R = 0.409\times(3.45)^3$

$R = 16.79$mm

Now,

$D = 2R = 2\times16.79 = 33.58$mm

Q) A closed coil helical spring of 10 cm mean dia is made up of 1 cm dia of rod and has 20 turns. The spring carries an axial load of 200N, determine the shearing stress. Take $C = 8.4 \times 10^4$ N/mm^2 and determine the deflection when carrying this load. Also calculate the stiffness of spring and the freq of free vibration for a mass hanging from it.

Sol:- Given data:-

$D = 10cm = 100mm$

$d = 1cm = 10mm$

$n = 20$

$W = 200N$

$C = 8.4 \times 10^4$ N/mm^2

i) w.k.t

$$\tau = \frac{16WR}{\pi d^3}$$

$$\tau = \frac{16 \times 200 \times 50}{\pi \times 10^3}$$

$$\tau = 50.92 \text{ N/mm}^2$$

ii) w.k.t

$$\delta = \frac{64WR^3n}{cd^4}$$

$$= \frac{64 \times 200 \times 50^3 \times 20}{8.4 \times 10^4 \times 10^4}$$

$$\delta = 38.09 \text{ mm}$$

iii) w.k.t

$$k = \frac{cd^4}{64R^3n}$$

$$= \frac{8.4 \times 10^4 \times 10^4}{64 \times 50^3 \times 20}$$

$$k = 5.25 \text{ N/mm}$$

iv)

$$\eta = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$\eta = \frac{1}{2\pi}\sqrt{\frac{9.81 \times 10^3}{38.09}}$$

$$\eta = 2.55 \text{ cycles/sec}$$

8) A closely coil helical spring made of 10mm dia steel wire has 15 coils of 100mm mean dia. The spring is subjected to an axial load of 100N, calculate i) Max shear stress induced ii) The deflection iii) Stiffness of the spring. Take $C = 8.16 \times 10^4 \ N/mm^2$.

Sol:- Given data :-

$d = 10mm$

$n = 15$

$D = 100mm$

$R = 50mm$

$W = 100N$

i) w.k.t

$$\tau = \frac{16WR}{\pi d^3}$$

$$\tau = \frac{16 \times 100 \times 50}{\pi \times 10^3}$$

$$\tau = 25.46 \ N/mm^2$$

ii) w.k.t

$$\delta = \frac{64WR^3 n}{cd^4}$$

$$\delta = \frac{64 \times 100 \times 50^3 \times 15}{8.16 \times 10^4 \times 10^4}$$

$$\delta = 14.70mm$$

iii) w.k.t

$$k = \frac{cd^4}{64R^3 n}$$

$$k = \frac{8.16 \times 10^4 \times 10^4}{64 \times 50^3 \times 15}$$

$$k = 6.8 \ N/mm$$

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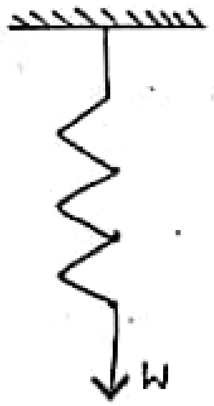
Expression for stresses in open coil helical spring:



Here, the actual load W causes the moment. As the pitch distance and helix angle is more in open coil helical spring, we get both bending and twisting. So, the moment which is formed due to load W is resolved into two components i.e twisting moment (T) along the spring axis and bending moment (M) along the axis $\perp^r$ to spring axis

w.k.t, so

$$T = WR\cos\alpha$$

$$M = WR\sin\alpha$$

w.k.t

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\theta = \frac{TL}{JC} = \frac{WR\cos\alpha \times L}{JC}$$

$$\theta = \frac{WR\cos\alpha \times L}{JC}$$

Now,

$$T = \frac{\pi}{16}\tau d^3$$

$$\tau = \frac{16T}{\pi d^3}$$

$$\boxed{\tau = \frac{16WR\cos\alpha}{\pi d^3}}$$

w.k.t

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{WR\sin\alpha \times \frac{d}{2}}{\frac{\pi}{64}d^4}$$

$$\sigma = \frac{WR\sin\alpha\, d}{2} \times \frac{64}{\pi d^4}$$

$$\boxed{\sigma = \frac{32WR\sin\alpha}{\pi d^3}}$$

Expression for δ

Work done against shear stress

$$W_1 = \frac{1}{2} \times T \times \theta$$

$$= \frac{1}{2} \times WR\cos\alpha \times \frac{WR\cos\alpha \times L}{JC}$$

$$W_1 = \frac{W^2R^2\cos^2\alpha\, L}{2JC}$$

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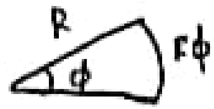
Work done against bending stress

$$W_2 = \frac{1}{2} \times M \times \phi$$

$$W_2 = \frac{1}{2} \times WR\sin\alpha \times \left[\frac{ML}{EI}\right]$$

$$W_2 = \frac{1}{2} \times WR\sin\alpha \times \frac{WR\sin\alpha L}{EI}$$

$$W_2 = \frac{W^2R^2\sin^2\alpha L}{2EI}$$



$$R\phi = L \Rightarrow R = L/\phi$$

Now,

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{\frac{L}{\phi}}$$

$$\frac{M}{I} = \frac{E\phi}{L}$$

$$\boxed{\phi = \frac{ML}{EI}}$$

Now,

Total Work done $= W_1 + W_2$

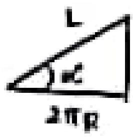
$$\frac{1}{2} \times W \times \delta = \frac{W^2R^2\cos^2\alpha L}{2JC} + \frac{W^2R^2\sin^2\alpha L}{2EI}$$

$$\delta = WR^2L\left[\frac{\cos^2\alpha}{JC} + \frac{\sin^2\alpha}{EI}\right] \quad \text{—} ①$$

But,

$$J = \frac{\pi}{32}d^4 \; ; \; I = \frac{\pi}{64}d^4$$

Now,



$$\cos\alpha = \frac{2\pi R}{L}$$

$$L = \frac{2\pi R}{\cos\alpha} \text{ (for one coil)}$$

For n coils total length

$$L = \frac{2\pi Rn}{\cos\alpha}$$

substitute these values in eqn ①

$$\delta = WR^2 \times \frac{2\pi Rn}{\cos\alpha}\left[\frac{\cos^2\alpha}{\frac{\pi}{32}d^4 \times C} + \frac{\sin^2\alpha}{\frac{\pi}{64}d^4 \times E}\right]$$

$$\delta = \frac{2\pi R^3Wn}{\cos\alpha}\left[\frac{32\cos^2\alpha}{\pi d^4C} + \frac{64\sin^2\alpha}{\pi d^4E}\right]$$

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$$\delta = \frac{64WR^3n}{\cos\alpha}\left[\frac{\cos^2\alpha}{Cd^4} + \frac{2\sin^2\alpha}{Ed^4}\right]$$

$$\boxed{\delta = \frac{64WR^3n}{d^4\cos\alpha}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]}$$

① The data related to open coil helical spring is given below: i) diameter of spring wire = 1.5cm ii) Mean coil dia = 15cm iii) No. of coils = 20 iv) helix angle = 12° v) Axial load = 800N. Take $C = 8\times10^6\ N/cm^2$ and $E = 2\times10^5\ N/mm^2$. Determine the intensity of Bending stress, shear stress, and axial deflection.

sol:-

$d = 1.5cm = 15mm$

$D = 15cm = 150mm$

$R = 75mm$

$n = 20$

$\alpha = 12^\circ$

$W = 800N$

$C = 8\times10^6\ N/cm^2 = 8\times10^4\ N/mm^2$

$E = 2\times10^5\ N/mm^2$

i) w.k.t

$$\sigma = \frac{32WR\sin\alpha}{\pi d^3}$$

$$\sigma = \frac{32\times800\times75\times\sin12^\circ}{\pi\times15^3}$$

$$\sigma = 37.64\ N/mm^2$$

ii) w.k.t

$$\tau = \frac{16WR\cos\alpha}{\pi d^3}$$

$$\tau = \frac{16\times800\times75\times\cos12^\circ}{\pi\times15^3}$$

$$\tau = 88.54\ N/mm^2$$

iii) w.k.t

$$\delta = \frac{64WR^3n}{d^4\cos\alpha}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$$

$$\delta = \frac{64\times800\times75^3\times20}{15^4\times\cos12^\circ}\left[\frac{\cos^2(12^\circ)}{8\times10^4} + \frac{2\times\sin^2(12^\circ)}{2\times10^5}\right]$$

$$\delta = 108.10\ mm$$

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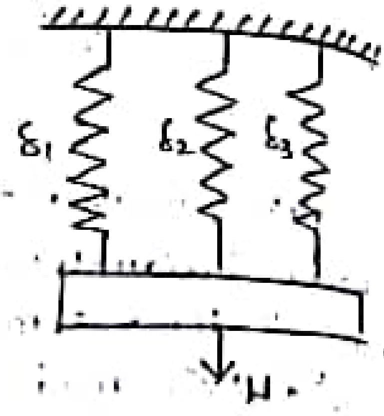
Expression for springs in parallel:

$$K = \frac{W}{\delta}$$



As δ is constant

$\delta_1 = \delta_2 = \delta_3 = \delta$

W is different for diff. springs

$$\therefore K = \frac{W}{\delta}$$

$$W = k\delta$$

$$W = W_1 + W_2 + W_3$$

$$K\delta = k_1\delta_1 + k_2\delta_2 + k_3\delta_3$$

$$K\delta = K_1\delta + K_2\delta + k_3\delta$$

$$\boxed{K = K_1 + K_2 + K_3}$$

Expression for springs in series:-

As W = constant

$W_1 = W_2 = W_3$

δ is diff for diff springs

$$K = \frac{W}{\delta}$$

$$\delta = W/K$$

$$\delta = \delta_1 + \delta_2 + \delta_3$$

$$\frac{W}{K} = \frac{W_1}{K_1} + \frac{W_2}{K_2} + \frac{W_3}{K_3}$$

$$\frac{W}{K} = \frac{W}{K_1} + \frac{W}{K_2} + \frac{W}{K_3}$$

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

$$1/K = \frac{K_2K_3 + K_1K_3 + K_1K_2}{K_1K_2K_3}$$

$$K = \frac{K_1K_2K_3}{K_2K_3 + K_1K_3 + K_1K_2}$$

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① Two springs connected in series carries a load of 2kN. one spring has 12 coils of 6mm wire bound in a dia of 30mm and 2nd spring has 16 coils of wire dia 8mm bound in a dia of 40mm. Find the stiffness k for composit system and Max stress produced in each wire. Take
$c = 8$ Gpa.

sol:- Given data

one spring: $n_1 = 12$

$d_1 = 6mm$

$D_1 = 30mm$

$R_1 = 15mm$

second spring: $n_2 = 16$

$d_2 = 8mm$

$D_2 = 40mm$

$R_2 = 20mm$

$W = 2kN = 2000 N$

$C = 8 \times 10^9 N/m^2 = 8 \times 10^4 N/mm^2$

1st spring

w.k.t

$$k_1 = \frac{cd_1^4}{64 R_1^3 n_1}$$

$$= \frac{8 \times 10^4 \times 6^4}{64 \times 15^3 \times 12}$$

$$k_1 = 40 N/mm$$

2nd spring

w.k.t

$$k_2 = \frac{cd_2^4}{64 R_2^3 n_2}$$

$$= \frac{8 \times 10^4 \times 8^4}{64 \times 20^3 \times 16}$$

$$k_2 = 40 N/mm$$

Stiffness $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$

$$k = \frac{40 \times 40}{40+40} = 20 N/mm$$

Now,

Max shear stress for 1st spring

$$\tau_1 = \frac{16 W R_1}{\pi d_1^3}$$

$$\tau_1 = \frac{16 \times 2000 \times 15}{\pi \times 6^3}$$

$$\tau_1 = 707.35 N/mm^2$$

Max shear stress for sec spring

$$\tau_2 = \frac{16 W R_2}{\pi d_2^3}$$

$$= \frac{16 \times 2000 \times 20}{\pi \times 8^3}$$

$$\tau_2 = 397.88 N/mm^2$$

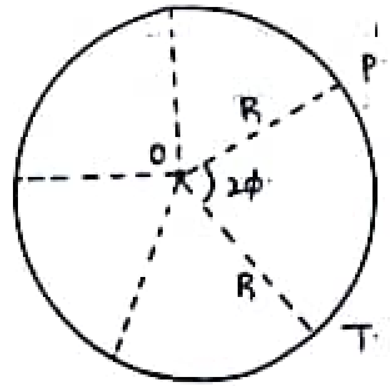
BEAMS CURVED IN PLAN

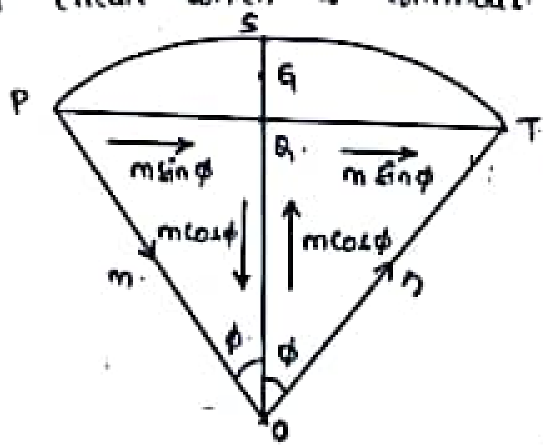
Introduction:

The analysis of beams which were represented by a straight line in plan and subjected to vertical loads was dealt previously. Those beams transfer the load by developing B.M and shear. Ocasionally beams which are curved in plan are also used. The common utility of these beams is to support walls of circular water tanks, silos and circular balcony slabs. In transferring loads, these beams develop not only B.M & shear but also torsion.

CIRCULAR BEAMS LOADED UNIFORMLY AND SUPPORTED ON SYMMETRICALLY PLACED COLUMNS:

Consider a continous circular beam which is supported symmetrically on columns. Let the beam is subjected to a uniformly distributed load 'W'. and beam forms a closed circuit which is continous.





As the circular beam is symmetrical, the vertical support reactions will be equal at each column. The twisting moment and shear force will be zero at the mid span. At the support ends the twisting moment will be zero. Consider a beam portion PT, which subtends an angle 2ϕ with radius R.

$\therefore$ Load on arc PT = Load x Length of the arc.

$$= W \times R(2\phi)$$

$$= 2WR\phi$$

Note:-

The C.G of an arc is given as

$$C.G = \frac{\text{Radius} \times \sin\left(\frac{\text{Angle}}{2}\right)}{\frac{\text{Angle}}{2}}.$$

Here.

In the fig.
$$\boxed{OG = \frac{R\sin\phi}{\phi}}$$

(a) <u>Shear Force and Bending Moment:</u>

The shear force and B.M will be the same at the support ends as the beam is symmetrical. If 'F' is the S.F and 'm' be the B.M @ the support then the S.F is given as.

$$F = \frac{2WR\phi}{2}$$

$$\boxed{F = WR\phi.}$$

The BM about the radial axis PO and OT will be hogging in nature at each support ends.
Hence the BM can be resolved in to two components along the axis of C.G. i.e.,

$m \sin\phi$ about PQ
$m \cos\phi$ about QO at support end P.

Similarly at support end T.

$m\sin\phi$ about QT
$m \cos\phi$ about QO can be resolved

Consider the chord PT, then the ~~end~~ total moments about their chord is $2m\sin\phi$.

$$m\sin\phi + m\sin\phi = 2m\sin\phi$$

Along the same chord PT, the moments due to external forces are $W \times GQ$

$$= W \times GQ$$
$$= 2WR\phi \times GQ$$
$$= 2WR\phi\left[GO - QO\right]$$
$$= 2WR\phi\left[\frac{R\sin\phi}{\phi} - R\cos\phi\right]$$

In Δ^{le} QOT, $\cos\phi = \frac{QO}{OT} \Rightarrow QO = OT\cos\phi$
$= R\cos\phi$

Now.

Total moments about the chord = Moments due to external forces.

$$\Rightarrow 2m\sin\phi = 2WR\phi \cdot \left[\frac{R\sin\phi}{\phi} - R\cos\phi\right]$$

$$m\sin\phi = WR\phi\left[\frac{R\sin\phi - \phi R\cos\phi}{\phi}\right]$$

$$m\sin\phi = WR^2(\sin\phi - \phi\cos\phi)$$

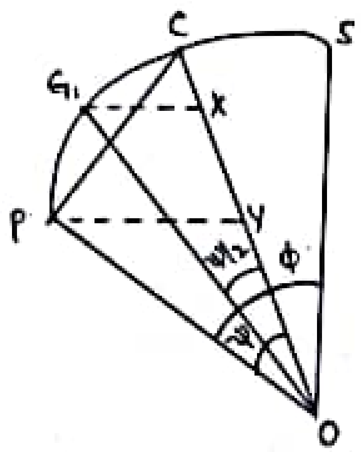
$$m = WR^2\left(\frac{\sin\phi}{\sin\phi} - \frac{\phi\cos\phi}{\sin\phi}\right)$$

$$m = WR^2(1 - \phi\cot\phi)$$

$$\therefore \boxed{m = WR^2(1 - \phi\cot\phi)}$$

Now.

Consider a point C which makes an angle ψ with PO. Let the S.F and B.M at this point be designated as F_ψ and m_ψ respectively



Now.

Support reaction at P = $WR\phi$ (S.F) at ψ

Now. Total load acting on PC = Load x Length of arc PC.

$$= W \times R(\psi)$$

$$\boxed{W_\psi = WR\psi}$$

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Now.

S.F @ C. $F_\psi = WR\phi - WR\psi$

$$\boxed{F_\psi = WR(\phi - \psi)}.$$

Now.

Let the CG on PC be G_1, which is given as.

$$G_1O = \frac{R \sin \psi/2}{\psi/2}.$$

Now.

From Δ^{le} G_1XO

$$\sin \psi/2 = \frac{G_1X}{G_1O}.$$

$$G_1X = G_1O \sin\psi/2$$

$$= \left(\frac{R \sin \psi/2}{\psi/2}\right) \sin \psi/2.$$

$$G_1X = \frac{R \sin^2 \psi/2}{\psi/2}.$$

Also.

$$\cos \psi/2 = \frac{XO}{G_1O}$$

$$XO = G_1O \cos \psi/2$$

$$= \left(\frac{R \sin \psi/2}{\psi/2}\right) \cos \psi/2.$$

$$= \frac{R \times 2 \sin \psi/2 \cos \psi/2}{\psi}.$$

$$XO = \frac{R \sin \psi}{\psi}.$$

Now

$CX = CO - XO = R - \frac{R\sin\psi}{\psi}$

$\Rightarrow CX = R\left[1 - \frac{\sin\psi}{\psi}\right]$

Now

$PY = R\sin\psi$

$CY = CO - YO = R - R\cos\psi = R(1-\cos\psi)$

Now

BM @ C = Moment about CO

$M\cos\psi$ will be a component of support end moment about CO and will be in hogging in nature

Now

Sagging moment at C = $F \times PY - W\psi \times G_1X - M\cos\psi$

$\Rightarrow M_\psi = (WR\phi \times R\sin\psi) - \left(WR\psi \times \frac{R\sin^2\psi/2}{\psi/2}\right) - WR^2(1-\phi\cot\phi)\cos\psi$

$= WR^2[\phi\sin\psi - 2\sin^2\psi/2 - \cos\psi + \phi\cot\phi\cos\psi]$

$\therefore \boxed{M_\psi = WR^2[\phi\sin\psi + \phi\cot\phi\cos\psi - 1]}$

$\left[\because 1 - 2\sin^2\frac{\theta}{2} = \cos\theta\right]$

__Twisting moment:__

The moment of all force at one side of support end about the tangential axis is called as twisting moment

Now.

let us denote the twisting moment @ point C as M_ψ^t

$$M_\psi^+ = -F \times CY + W_\psi \times CX + m\sin\psi$$

$$= m\sin\psi - F \times CY + W_\psi \times CX$$

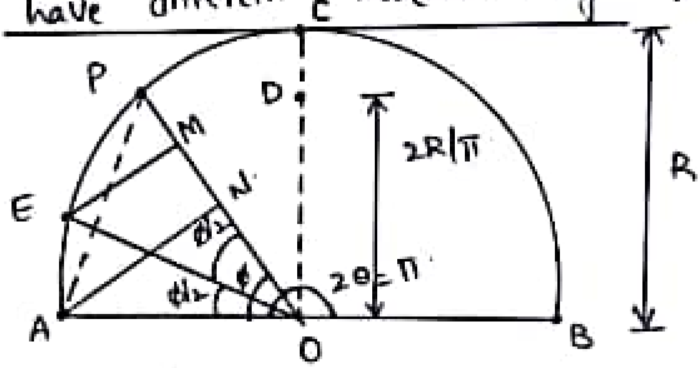
$$= \left[WR^2(1-\phi\cot\phi)\sin\psi\right] - \left[WR^2\phi \times R(1-\cos\psi)\right] + \left[WR\psi \times R\left(1-\frac{\sin\psi}{\psi}\right)\right].$$

$$M_\psi^+ = \left\{WR^2\left(\sin\psi - \phi\sin\psi \times \frac{\cos\phi}{\sin\phi}\right)\right\} - \left\{WR^2\phi(1-\cos\psi)\right\} + \left\{WR^2\psi\left(\psi - \frac{\sin\psi}{\psi}\right)\right\}$$

$$\boxed{M_\psi^+ = WR^2\{\phi\cos\psi - \phi\cot\phi\sin\psi - (\phi-\psi)\}}$$

SEMI CIRCULAR BEAM SIMPLY SUPPORTED ON THREE EQUALLY SPACED SUPPORTS :-

A semi circular beam simply supported on three equispaced columns ABC is shown in the figure. Let a uniformly distributed load w/unit length be applied on the beam. Reactions will be developed at all the supports A, B & C. The reaction components at A and B will be same (say R_1) due to symmetry and the support C will have different reaction (say R_2)



The total UDL is assumed to be acted upon the centroid D of the semi-circle as shown.

1. Reactions at supports :-

Total load = Load × length of AB.

$$= w \times \pi R.$$

$$W = w \cdot \pi R.$$

Dist. of C.G i.e., DO = $\dfrac{R \sin\left(\frac{Angle}{2}\right)}{\frac{Angle}{2}} = \dfrac{R \sin\frac{\pi}{2}}{\frac{\pi}{2}}$.

$$DO = \frac{2R}{\pi}.$$

Now,

$\Sigma Y = 0$.

$$R_1 + R_2 + R_1 = \omega \pi R$$

$$\Rightarrow \quad 2R_1 + R_2 = \omega \pi R$$

Now,

Equating the moments of all forces from tgt at C to zero.

$$2R_1 \times CO = \omega \times DC$$

$$2R_1 \times R = \omega \pi R \left(CO - DO \right)$$

$$2R_1 \times R = \omega \pi R \left(R - \frac{2R}{\pi} \right).$$

$$R_1 = \frac{\omega \pi R^2}{2R} \left(\frac{\pi - 2}{\pi} \right).$$

$$R_1 = \frac{\omega R}{2} (\pi - 2).$$

Now,

$$R_2 = \omega \pi R - 2R_1 = \omega \pi R - 2\left(\frac{\omega R}{2} (\pi - 2) \right)$$

$$R_2 = \omega \pi R - \omega \pi R + 2\omega R.$$

$$\therefore \quad R_2 = 2\omega R.$$

Now,

Consider any point 'P' on the beam at an inclination of 'ϕ' from OA.

Now,

$$C.G \text{ of load } EO = \frac{R \sin \phi/2}{\phi/2}.$$

Now.

Total load acting on AP. $W\phi = wR\phi$.

Draw the $\perp^{ler}$ lines EM & AN from the radial line OP to the points E and A.

Now.

From Δ^{le} ANO. $\sin\phi = \frac{AN}{AO}$.

$\Rightarrow AN = AO \sin\phi$

$AN = R\sin\phi$.

$\cos\phi = \frac{ON}{AO}$

$\Rightarrow ON = AO\cos\phi = R\cos\phi$.

Now.

From Δ^{le} EMO. $\sin\phi/2 = \frac{EM}{EO}$

$\Rightarrow EM = EO \sin\phi/2$

$$= \left[\frac{R\sin\phi/2}{\phi/2}\right]\sin\phi/2 = \frac{R\sin^2\phi/2}{\phi/2}$$

$\cos\phi/2 = \frac{OM}{EO}$.

$\Rightarrow OM = EO\cos\phi/2$.

$$= \left[\frac{R\sin\phi/2}{\phi/2}\right]\cos\phi/2 = \frac{R\sin\phi}{\phi}$$

2. <u>S.F at P:-</u>

S.F at P is given by.

$F_\phi = R_1 - wR\phi$.

$= \frac{WR}{2}(\pi-2) - (WR\phi) = WR\left(\frac{\pi}{2} - \frac{2}{2} - \phi\right].$

$\therefore \; F_\phi = WR\left[\frac{\pi}{2} - 1 - \phi\right]$

At point C, $\phi = \pi/2$

$\therefore \; F_\phi = F_c = WR\left[\frac{\pi}{2} - 1 - \frac{\pi}{2}\right].$

$F_\phi = F_c = -WR.$

3. <u>Bending moment at any point :-</u>

B.M at point 'P' is equal to the moment of all the forces to the left or right of point P. about pt. P.

$M_\phi = R_1 \times AN - W_\phi \times EM.$

$= \frac{WR}{2}(\pi-2) \times R\sin\phi - WR\phi \times R\frac{\sin^2\phi/2}{\phi/2}$

$M_\phi = WR^2\left[\frac{\pi-2}{2}\sin\phi - 2\sin^2\phi/2\right]. \rightarrow (1)$

Now.

At A, $\phi = 0 \Rightarrow M_\phi = M_A = 0.$

At C, $\phi = \pi/2 \Rightarrow M_\phi = M_c.$

$= WR^2\left[\frac{\pi-2}{2}\sin\frac{\pi}{2} - 2\sin^2\frac{\pi}{4}\right].$

$= WR^2\left[\frac{\pi-2}{2} \times 1 - 2 \times \frac{1}{2}\right].$

$= WR^2\left[\frac{(\pi-2)}{2} - 1\right].$

$$= \omega R^2 \left[\frac{(\pi-2)-2}{2} \right].$$

$$= \omega R^2 \left[\frac{\pi-4}{2} \right]$$

$\therefore$ $M_c = -0.429\, \omega R^2$ (hogging moment)

For max. sagging moment in the length AC of the beam.

$$\frac{dM_\phi}{d\phi} = 0.$$

$$\Rightarrow \omega R^2 \left[\frac{\pi-2}{2} \cos\phi - 2 \times 2 \sin(\phi/2) \times \cos\phi/2 \times \frac{1}{2} \right] = 0.$$

$$\frac{\pi-2}{2} \times \cos\phi - 2 \sin(\phi/2) \cos(\phi/2) = 0.$$

$$\frac{\pi-2}{2} \cos\phi = \sin\phi.$$

$$\tan\phi = \frac{\pi-2}{2} = 0.571$$

$$\Rightarrow \phi = \tan^{-1}(0.571) = 29.718^\circ = 29^\circ 44'$$

$\therefore$ $\phi = 29^\circ 44'$ (or) 0.519 radians.

Substituting ϕ in ① we get M_{max}.

$$\Rightarrow M_{max} = \omega R^2 \left[\frac{\pi-2}{2} \sin(29.718^\circ) - 2 \sin^2 \frac{29.718}{2} \right].$$

$M_{max} = 0.151\, \omega R^2$ (sagging)

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for point of contraflexure, equate $M\phi = 0$.

$$wR^2\left[\frac{\pi-2}{2}\sin\phi - 2\sin^2\phi/2\right] = 0.$$

$$\frac{\pi-2}{2}\sin\phi = 2\sin^2(\phi/2)$$

$$\Rightarrow \left(\frac{\pi-2}{2}\right) 2\sin\phi/2 \cos\phi/2 = 2\sin^2\phi/2.$$

$$\left(\frac{\pi-2}{2}\right) 2\cos\phi/2 = 2\sin(\phi/2)$$

$$\tan(\phi/2) = \frac{\pi-2}{2} = 0.571.$$

$$\Rightarrow \phi/2 = \tan^{-1}(0.571) = 29.718^\circ.$$

$$\therefore \phi = 59.435^\circ = 1.037 \text{ radians.}$$

<u>Torsional moment at any point 'P'</u> :-

$$M^t_\phi = R_1 \times PN - W_\phi \times PM.$$

$$= \frac{WR}{2}(\pi-2) \times (PO - NO) - W_\phi \times (PO - MO)$$

$$= \frac{WR}{2}(\pi-2) \times (R - R\cos\phi) - WR\phi\left(R - \frac{R\sin\phi}{\phi}\right).$$

$$M^t_\phi = WR^2\left[\frac{\pi-2}{2} - \frac{\pi-2}{2}\cos\phi - \phi + \sin\phi\right].$$

The torsional moment will be max. at the point of contraflexure i.e., at $\phi = 59.435^\circ = 1.037$ radians

$$M^t_{max} = WR^2\left[\frac{\pi-2}{2} - \frac{\pi-2}{2}\cos(59.435) - 1.037 + \sin(59.435)\right]$$

$$\therefore M^t_{max} = 0.105\, WR^2.$$

Also torsional moment will be zero at every support

Q6. **A curved beam semi-circular in plan, 5 m radius, and supported on three equally spaced supports. The beam-carries a uniformly distributed load of 20 kN/m of the circular length. Analyse the beam and draw the twisting moment diagram.**

Answer : **April-11, Set-2, Q6 M[15]**

Given that,

Radius of semi-circle, $R = 5$ m

Uniformly distributed load, $W = 20$ kN/m

Reactions

$$R_A = \frac{WR}{2}(\pi - 2)$$

$\frac{wR}{2}(\pi-2$ [illegible]

$$= \frac{20 \times 5}{2}(\pi - 2)$$

$$\boxed{R_A = 57.07 \text{ kN}} \Rightarrow$$

$R_A = R_B = 57.07$ kN.

$$R_C = 2WR$$
$$= 2 \times 20 \times 5$$

$$\boxed{R_C = 200 \text{ kN}}$$

Shear Force

Shear force at A, $SF_A = R_A$

$$\boxed{SF_A = 57.07 \text{ kN}}$$

Shear force at C, $SF_c = -WR$

$$= -20 \times 5$$

$$\boxed{SF_C = -100 \text{ kN}}$$

Shear force at any other position,

$$SF_\phi = wR\left[\frac{\pi}{2} - 1 - \phi\right]$$

$$= 20 \times 5\left[\frac{\pi}{2} - 1 - \phi \times \frac{\pi}{180}\right]$$

$$SF_\phi = 100[0.5708 - 0.01745\phi] \quad \text{...(1)}$$

at $\phi = 90°$

$$SF_\phi = 100[0.5708 - 0.01745 \times 90]$$
$$SF_\phi = -99.97 \text{ kN}$$

Now, equating equation (1) to zero.

$$SF_{\phi} = 0$$

$$100[0.5708 - 0.01745\phi] = 0$$

$$0.5708 - 0.01745\phi = 0$$

$$\phi = \frac{0.5708}{0.01745}$$

$$\phi = 32.71°$$

Bending Moment

Bending moment at C,

$$M_c = -0.429\,WR^2$$

$$= -0.429 \times 20 \times (5)^2$$

$$M_c = -214.5\,\text{kNm (hogging)}$$

Maximum bending moment,

$$M_{max} = 0.1514\,WR^2$$

$$= 0.1514 \times 20 \times (5)^2$$

$$M_{max} = 75.7\,\text{kNm} \quad \text{(sagging)}$$

Bending moment at any other position,

$$M_{\phi} = WR^2[0.5708 \sin\phi - 2\sin^2\phi/2]$$

$$= 20 \times 5^2[0.5708 \sin\phi - 2\sin^2\phi/2]$$

$$M_{\phi} = 500[0.5708 \sin\phi - 2\sin^2\phi/2] \quad \text{...(2)}$$

The following are the values of bending moment at 30° interval.

Angle (ϕ)	Bending moment at any other position (M_{ϕ})	Position
0	0.00	End
29.72°	75.719	Maximum bending moment sagging
30°	75.712	
59.44°	0.00	Torsion point
60°	−2.83	
90°	−214.6	Maximum bending moment hogging

To locate the position, where BM is zero, equating equation (2) to zero.

$$500[0.5708 \sin\phi - 2\sin^2\phi/2] = 0$$

$$0.5708 \sin\phi - 2\sin^2\phi/2 = 0$$

$$0.5708 \sin\phi = 2\sin^2\phi/2$$

We know that,

$$\boxed{\sin\theta = 2\sin\theta/2\cos\theta/2}$$

$$\therefore\ 0.5708 \times 2\sin\phi/2\cos\phi/2 = 2\sin^2\phi/2$$

$$0.5708\cos\phi/2 = \sin\phi/2$$

$$\frac{\sin\phi/2}{\cos\phi/2} = 0.5708$$

$$\tan\phi/2 = 0.5708$$
$$\phi/2 = \tan^{-1}(0.5708)$$
$$\phi/2 = 29.72$$
$$\boxed{\phi = 59.44^{\circ}}$$

∴ The torsional moment point is also the point of contra-flexure.

Torsional Moment

Maximum torsional moment,

$$M_m^t = 0.1045\, WR^2$$
$$= 0.1045 \times 20 \times (5)^2$$
$$= 52.25 \text{ kN-m}$$

at $\phi = 59.44^{\circ}$

The torsional moment distribution is given as,

$$M_\phi^t = WR^2\left[\frac{\pi-2}{2} - \frac{\pi-2}{2}\cos\phi - \phi + \sin\phi\right]$$

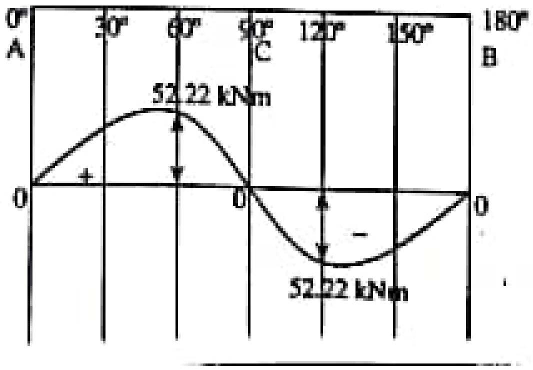
$$= 20(5)^2\left[\frac{\pi-2}{2} - \frac{\pi-2}{2}\cos\phi + \sin\phi - \phi\times\frac{\pi}{180}\right]$$

$$= 500[0.5708 - 0.5708\cos\phi + \sin\phi - 0.01745\phi]$$

The following are the values of torsional moment at 30° interval.

Angle (ϕ)	Torsional moment at any other position (M_ϕ^t)	Position
0	0.00	End of beam
30	26.48	
59.44°	52.22	Zero bending moment point
60°	52.21	
90°	0.00	Centre of beam

Twisting Moment Diagram



MODULE-II

DIRECT AND BENDING STRESSES

UNIT-III DIRECT AND BENDING STRESSES

* **Direct stress:-**

Direct stress produced in a body when it is subjected to an axial tensile or compressive load.

* **Bending stress:-**

Bending stress produced in a body when it is subjected to bending moment.

But if a body is subjected to both axial load and bending moment then both stresses will be produced in a body i.e direct & Bending stresses. Both these stresses act normal to the cross section. Hence, two stresses may be added into a single resultant stress.

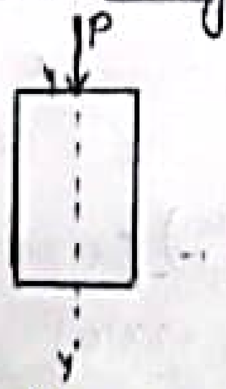
Eccentric load or Eccentricity:-

Eccentric load is the load whose line of action doesn't coincide with the axis of column. The load may be eccentric to one of the axis or about both the axis.

Due to eccentric load both direct & bending stresses will be produced.

Combined bending & direct stress:-

i)



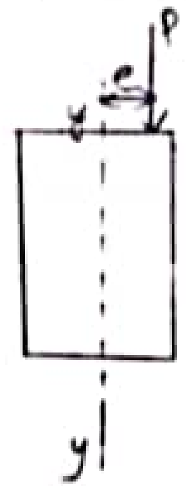
Consider a case of a column subjected to a compressive load 'p' along the axis of column. This load will cause a direct compressive stress whose intensity will be uniform across the cross-section of the column. So, stress acting on column in this case is only the direct stress and it is given by $\boxed{\sigma_d = \frac{P}{A}}$

where σ_d = Intensity of stress.

P = load acting on the column.

A = Area of cross-section.

ii)



Now consider a case of a column subjected to compressive load 'P' whose line of action is at a distance of 'e' from the axis of the column as shown in the figure.

Here, e is the eccentricity of the load.

This eccentric load causes both direct & bending stresses.

* Resultant stresses when a column of rectangular section is subject to an eccentric load.

A column of rectangular section subjected to an eccentric load as shown in fig (1.a). let load is eccentric w.r.t. to yy axis as shown in fig (1.b). It is mentioned at eccentric load causes direct as well as bending stresses.

let P = eccentric load on column.

e = eccentricity of the load.

σ_d = direct stress.

σ_b = bending stress

b = width of column

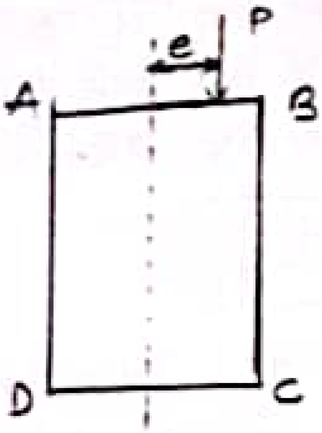


fig (1.a)

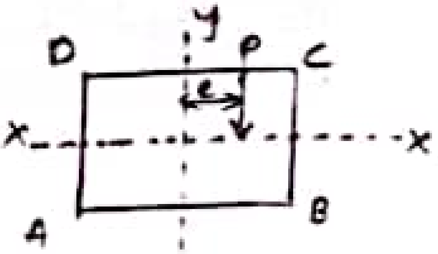


fig (1.b)

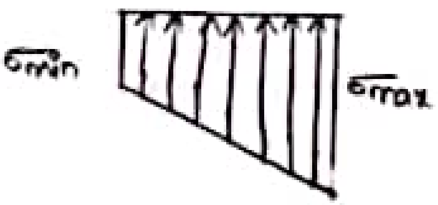


fig (1.c)

Now, $\sigma_d = \frac{P}{A}$.

Bending stress at any point of the column section at a distance 'y' from the neutral axis is given by

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \pm \frac{My}{I}$$

where $y = \pm b/2$

$$I = \frac{db^3}{12}$$

$$M = Pe$$

The stress will be maximum along the layer BC and minimum along the layer AD.

Now, $\sigma_{max} = \sigma_d + \sigma_b$

$$= \frac{P}{A} + \frac{My}{I}$$

$$= \frac{P}{A} + \frac{Pe\, b/2}{\frac{db^3}{12}}$$

$$= \frac{P}{A} + \frac{Peb}{\cancel{2}} \times \frac{\cancel{12}^{6}}{db^{\cancel{3}2}}$$

$$= \frac{P}{A} + \frac{6Pe}{db^2}$$

$$= \frac{P}{A} + \frac{6Pe}{bd\, b}$$

$$= \frac{P}{A} + \frac{6Pe}{Ab}$$

$$= \frac{P}{A} + \frac{P}{A}\frac{6e}{b}$$

$$\boxed{\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]}$$

Now, $\sigma_{min} = \sigma_d - \sigma_b$

$$= \frac{P}{A} - \frac{My}{I}$$

$$= \frac{P}{A} - \frac{Pe \times \frac{b}{2}}{\frac{db^3}{12}}$$

$$= \frac{P}{A} - \frac{Pe\cancel{b}}{\cancel{2}} \times \frac{\cancel{12}^{6}}{db^{\cancel{3}2}}$$

$$= \frac{P}{A} - \frac{Pe\,6}{db^2}$$

$$= \frac{P}{A} - \frac{Pe6}{bd\,b}$$

$$= \frac{P}{A} - \frac{6Pe}{Ab}$$

$$= \frac{P}{A} - \frac{P}{A}\frac{6e}{b}$$

$$\boxed{\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right]}$$

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Stresses are shown in fig (1.c). the Resultant stress along the width of the column may vary may a straight line.

* **Note:-**

1) If σ_{min} is '-ve', then stresses along AD will be Tensile stresses.
2) If σ_{min} is 0, then there will be no stresses (i.e no compr. or tensile stress along AD).
3) If σ_{min} is '+ve', then stresses along AD will be compressive stresses.

1) A rectangular column of width 200mm and thickness 150mm carries a point load of 240 kN at an eccentricity of 10mm.
   i) determine maximum & minimum stresses of section.
   ii) If the minimum stress on the section is zero, then find the eccentricity of the point load of 240kN acting on the rectangular column. Also determine the corresponding maximum stress on the section.
   iii) If eccentricity is given 50mm instead of 30mm then find the maximum and minimum stresses on the section. Also plot this stresses along the width of the section.

Given:- $b = 200$ mm

$d = 150$ mm

$P = 240$ kN $= 240 \times 10^3$ N

i) $e = 10$ mm

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$$

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$$\sigma_{max} = \frac{240 \times 10^3}{200 \times 150}\left[1 + \frac{6 \times 10}{200}\right].$$

$$\sigma_{max} = 10.4 \ N/mm^2$$

w.k.t

$$\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{240 \times 10^3}{200 \times 150}\left[1 - \frac{6 \times 10}{200}\right]$$

$$\sigma_{min} = 5.6 \ N/mm^2$$

ii) $\sigma_{min} = 0$

$$\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right]$$

$$0 = \frac{240 \times 10^3}{200 \times 150}\left[1 - \frac{6 \times e}{200}\right]$$

$$1 = \frac{6e}{200}$$

$$e = \frac{200}{6} = 33.33 \ mm$$

Now, $\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$

$$= \frac{240 \times 10^3}{200 \times 150}\left[1 + \frac{6 \times 33.33}{200}\right]$$

$$\sigma_{max} = 15.99 \ N/mm^2$$

iii) $e = 50 mm$

$$\sigma_{max} = \frac{240 \times 10^3}{200 \times 150}\left[1 + \frac{6 \times 50}{200}\right]$$

$$\sigma_{max} = 20 N/mm^2$$

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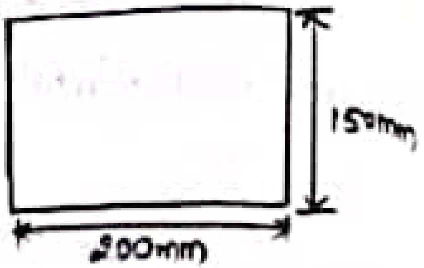
$$\sigma_{min} = \frac{240\times10^3}{200\times150}\left[1-\frac{6\times50}{200}\right]$$

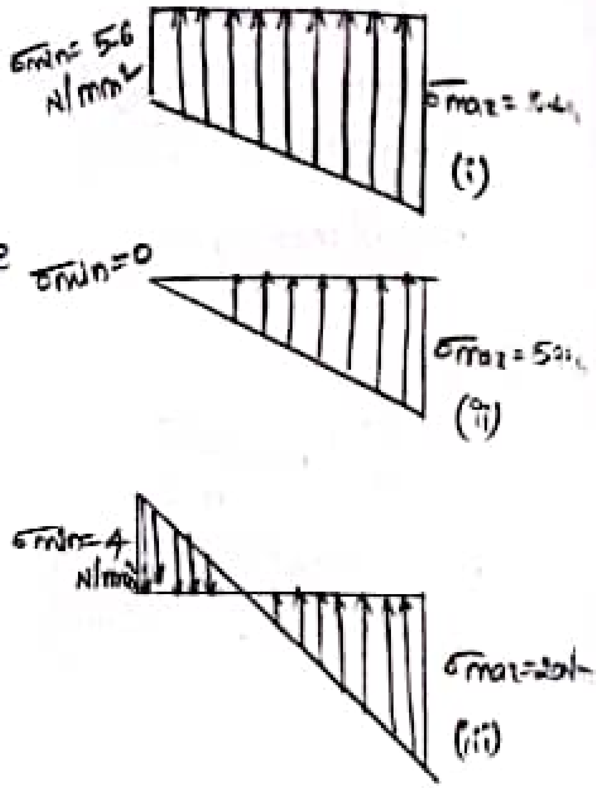
$$\sigma_{min} = -4\ N/mm^2$$

Stress diagram



from above equations we can say that

i) $\sigma_{min}=0$, when $e=\frac{b}{6}$.
there is no tensile or compressive stress in this case.

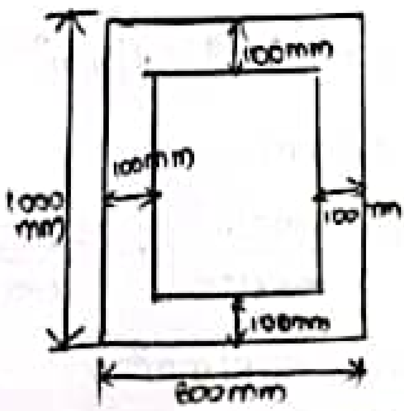


ii) $\sigma_{min} = +ve$, when $e < b/6$
the stress here is compressive stress.

iii) $\sigma_{min} = -ve$, when $e > b/6$
the stress here is tensile stress.

2) A hollow rectangular column of 1m length and external width of 0.8 m is 10 cm thick. calculate the maximum and minimum stress in the section of a column if vertical load of 200 kN is acting with an eccentricity of 15 cm.

Soln:- $l=1m= 1000mm$
$B= 0.8m= 800\ mm$
$t= 10cm= 100mm$
$P= 200kN= 200\times10^3\ N$
$e = 15cm = 150mm$



w.k.t

$\sigma_{max} = \sigma_d + \sigma_b$

$\sigma_d = P/A$

But $A = BD - bd$

But

$b = B - 2t = 800 - 200 = 600mm$

$d = D - 2t = 1000 - 200 = 800mm$

$A = 800 \times 1000 - 800 \times 600$

$A = 320000 \ mm^2$

$I = \cancel{BD} \frac{DB^3}{12} - \frac{db^3}{12}$

$= \frac{1000 \times 800^3}{12} - \frac{800 \times 600^3}{12}$

$I = 2.82 \times 10^{10} \ mm^4$

$y = \frac{B}{2} = \frac{800}{2} = 400 \ mm$

$M = Pe$

$= 200 \times 10^3 \times 150$

$M = 300 \times 10^5 \ N\text{-}mm$

Now

$\sigma_d = \frac{P}{A} = \frac{200 \times 10^3}{320000}$

$\sigma_d = 0.625 \ N/mm^2$

Now, $\sigma_b = \frac{My}{I}$

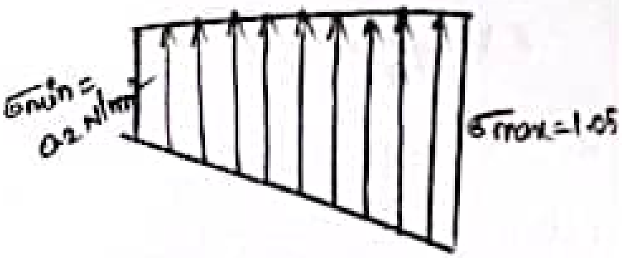
$= \frac{300 \times 10^5 \times 400}{2.82 \times 10^{10}}$

$\sigma_b = 0.425 \ N/mm^2$

Now, $\sigma_{max} = \sigma_d + \sigma_b$

$= 0.625 + 0.425$

$\sigma_{max} = 1.05 \ N/mm^2$



$\sigma_{min} = \sigma_d - \sigma_b$

$= 0.625 - 0.425$

$\sigma_{min} = 0.2 \ N/mm^2$

3) A short column of external diameter 40cm and internal diameter 20cm carries an eccentric load of 80kN. Find the greatest eccentricity which the load can have without producing tension on the cross-section.

Soln:-

Given:-

$D = 40cm = 400mm$

$d = 20cm = 200 \ mm$

$P = 80kN = 80 \times 10^{-3} \ N.$

$\sigma_{min} = 0$

W.K.T

$\sigma_{max} = \sigma_d + \sigma_b$

$\sigma_{min} = \sigma_d - \sigma_b$

But $\sigma_d = \frac{P}{A}$

Now, $A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(400^2 - 200^2)$

$A = 94247.77 \ mm^2$

Now, $\sigma_b = \frac{My}{I}$

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$= \frac{Pey}{I}$ $\quad \sigma_b = \frac{Pey}{I}$

But $I = \frac{\pi}{64}(D^4 - d^4)$

$I = \frac{\pi}{64}[400^4 - 200^4]$

$I = 1178097245 \text{ mm}^4.$

Now, $\sigma_b = \frac{80 \times 10^3 \times e \times 200}{1178097245}$

$\sigma_b = 0.0135e.$

Now, $\sigma_{min} = \sigma_d - \sigma_b$

$= \frac{P}{A} - \sigma_b$

$0 = \frac{80 \times 10^3}{94247.77} - 0.0135e$

$0.0135e = 0.848$

$e = \frac{0.848}{0.0135}$

$e = 62.81 \text{ mm}$

Now,

$\sigma_{max} = \sigma_d + \sigma_b$

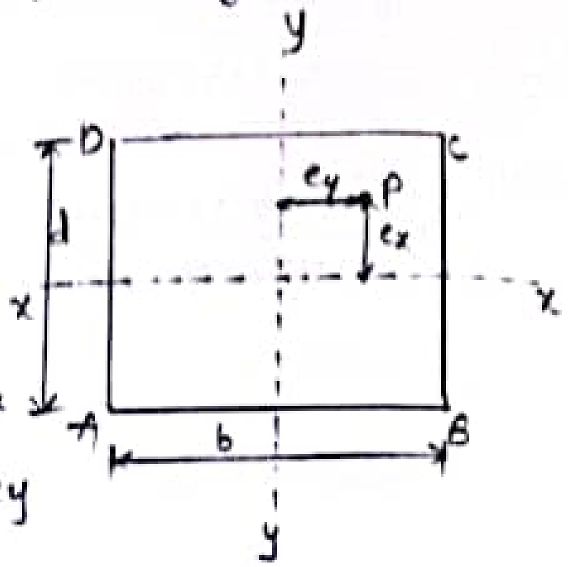
$= 0.848 + 0.0135e$

$\sigma_{max} = 0.848 + 0.0135 \times 62.81$

$\sigma_{max} = 1.69 \text{ N/mm}^2$ //

Resultant stress when of rectangular column is subjected to a load which of which is eccentric to both the axis.

A column of rectangular section ABCD subjected to a load which is eccentric to both the axis as shown in the figure.



Let P = eccentric load

e_x = eccentricity from x-x axis.

e_y = eccentricity from y-y axis.

M_x = Moment about x-x axis = Pe_x

M_y = Momment about y-y axis = Pe_y

b = width of column

d = depth of column

σ_{bx} = Bending stresses due to e_x

σ_{by} = Bending " " to e_y.

I_{xx} = MOI about x-x = $\frac{bd^3}{12}$

I_{yy} = M.O.I about yy = $\frac{db^3}{12}$

w.k.t

$$\sigma = \sigma_d + \sigma_b$$

But $\sigma_d = \frac{P}{A}$

Now $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{My}{I}$$

Now, $\sigma_{bx} = \frac{M_x \times y}{I_{xx}}$

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$$\sigma_{bx} = \frac{P e_x y}{I_{xx}}$$

Now, $$\sigma_{by} = \frac{M_y \times x}{I_{yy}}$$

$$\sigma_{by} = \frac{P e_y x}{I_{yy}}$$

Now

$$\sigma_R = \sigma_d \pm \sigma_b$$

$$\sigma_R = \sigma_d \pm \sigma_{bx} \pm \sigma_{by}$$

$$\boxed{\sigma_R = \frac{P}{A} \pm \frac{P e_x y}{I_{xx}} \pm \frac{P e_y x}{I_{yy}}}$$

1) At point C the coordinates of x & y are positive, hence the resultant stress will be maximum and it is given as

$$\boxed{\sigma_C = \frac{P}{A} + \frac{P e_x y}{I_{xx}} + \frac{P e_y x}{I_{yy}}}$$

2) At point A the coordinates of x & y are negative and hence the resultant stress will be minimum.

ie $$\boxed{\sigma_A = \frac{P}{A} - \frac{P e_x y}{I_{xx}} - \frac{P e_y x}{I_{yy}}}$$

3) At point D, x is -ive and y is +ive and hence the resultant stress will be

$$\sigma_D = \frac{P}{A} + \frac{P e_x y}{I_{xx}} - \frac{P e_y x}{I_{yy}}.$$

4) At point B, x is +ve and y is -ve and hence the resultant stress will be

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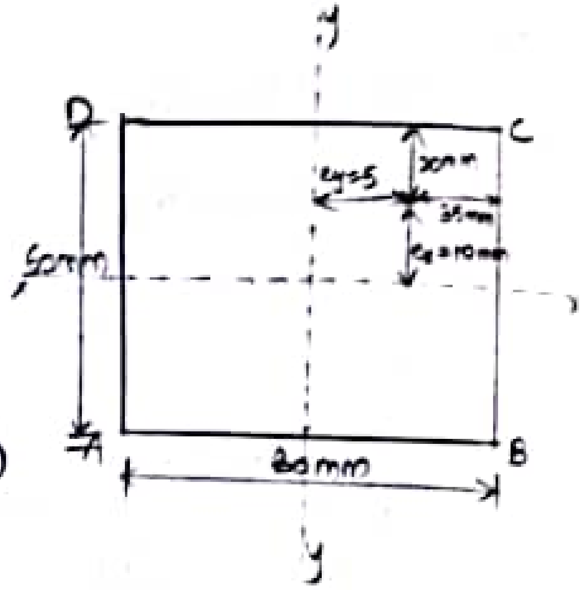
$$\boxed{\sigma_B = \frac{P}{A} - \frac{Pe_x y}{I_{xx}} + \frac{Pe_y x}{I_{yy}}}$$

1) A short column of rectangular section of 80mm×60mm carries a load of 40kN at a point 20 mm from the longer side and 35mm from the shorter side. Determine the maximum compressive and Tensile stresses in the section.



Soln: $P = 40kN$

from fig

$e_x = 10mm$

$e_y = 5\ mm$

W.k.t

$$\sigma = \frac{P}{A} + \frac{Pe_x y}{I_{xx}} + \frac{Pe_y}{I_{yy}} \quad —①$$

But $A = 80 \times 60 = 4800\ mm^2$

$I_{xx} = \frac{bd^3}{12} = \frac{80 \times 60^3}{12} = 1440000\ mm^4$

$I_{yy} = \frac{db^3}{12} = \frac{60 \times 80^3}{12} = 2560000\ mm^4$

$x = \pm 40mm$

$y = \pm 30mm$

Now, In order to get ~~more~~ max. compressive stress put $x = 40mm$, $y = 30mm$ in ①

Now,

$$\sigma_c = \frac{40 \times 10^3}{4800} + \frac{40 \times 10^3 \times 10 \times 30}{1440000} + \frac{40 \times 10^3 \times 5 \times 40}{2560000}$$

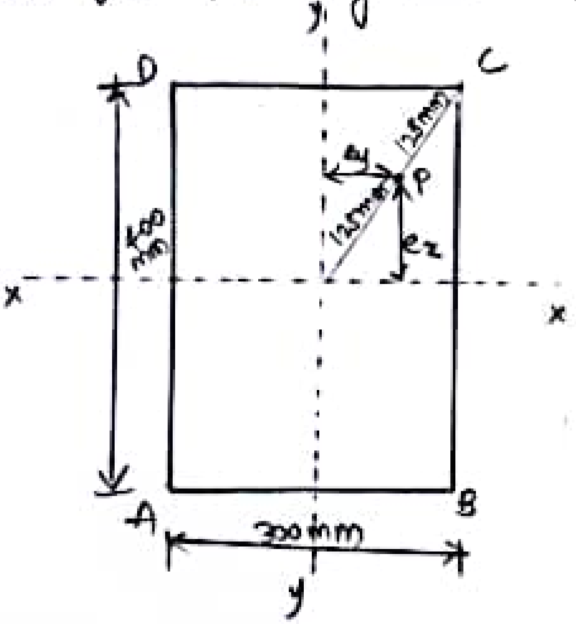
$\sigma_c = 19.79\ N/mm^2$ (compressive)

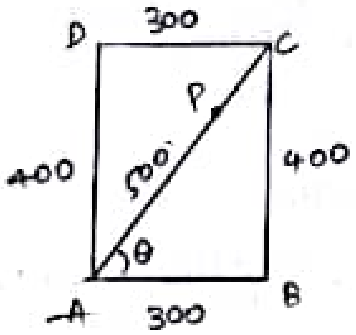
Now, In order to get max. tensile stress put $x = -40mm$ & $y = -30mm$ in eqn ①

Now
$$\sigma_A = \frac{40\times10^3}{4800} - \frac{40\times10^3\times10\times30}{1440000} - \frac{40\times10^3\times5\times40}{2560000}$$

$$\sigma_A = -3.125 \ N/mm^2 \text{ (tensile)}$$

2) A column is rectangular in cross section of 300 mm×400 mm in dimensions. The column carries an eccentric point load of 360 kN on one diagonal at a distance of quarter diagonal length from a corner. Calculate the stresses at all the four corners and draw the stress distribution diagram for any two adjacent sides.





Soln:-

Now
p will be @ a dist of
$\frac{1}{4}(500) = 125mm$ from c

$$\tan\theta = \frac{400}{300} = \frac{4}{3}$$

$$\cos\theta = \frac{300}{500} = \frac{3}{5}$$

$$\sin\theta = \frac{400}{500} = \frac{4}{5}$$

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$\sin\theta = \frac{e_x}{125}$

$e_x = 125\sin\theta$

$= 125 \times \frac{4}{5}$

$e_x = 100\,mm$

$\cos\theta = \frac{e_y}{125}$

$e_y = 125\cos\theta$

$= 125 \times \frac{3}{5}$

$e_y = 75\,mm$

$\therefore e_x = 100\,mm$

$e_y = 75\,mm$

w.k.t

$\sigma = \frac{P}{A} + \frac{P e_x y}{I_{xx}} + \frac{P e_y x}{I_{yy}}$

But

$A = 300 \times 400 = 120000\,mm^2$

$I_{xx} = \frac{bd^3}{12} = \frac{300 \times 400^3}{12} = 16 \times 10^8\,mm^4$

$I_{yy} = \frac{db^3}{12} = \frac{400 \times 300^3}{12} = 9 \times 10^8\,mm^4$

$x = \pm 150\,mm$

$y = \pm 200\,mm$

Now, stress @ point A.

Stress Here $x = -150mm, y = -200mm$

$\sigma_A = \frac{360 \times 10^3}{120000} - \frac{360 \times 10^3 \times 100 \times 200}{16 \times 10^8} - \frac{360 \times 10^3 \times 75 \times 150}{9 \times 10^8}$

$\boxed{\sigma_A = -6\,N/mm^2}$ (Tensile)

stress @ pt B

$x = 150mm, \quad y = -200mm$

$$\sigma_B = \frac{360\times10^3}{120000} - \frac{360\times10^3\times100\times200}{16\times10^8} + \frac{360\times10^3\times75\times150}{9\times10^8}.$$

$\boxed{\sigma_B = 3 N/mm^2}$ (compressive)

Stress @ pt C:-

$x = 150$ mm, $y = 200$ mm

$$\sigma_C = \frac{360\times10^3}{120000} + \frac{360\times10^3\times100\times200}{16\times10^8} + \frac{360\times10^3\times75\times150}{9\times10^8}.$$

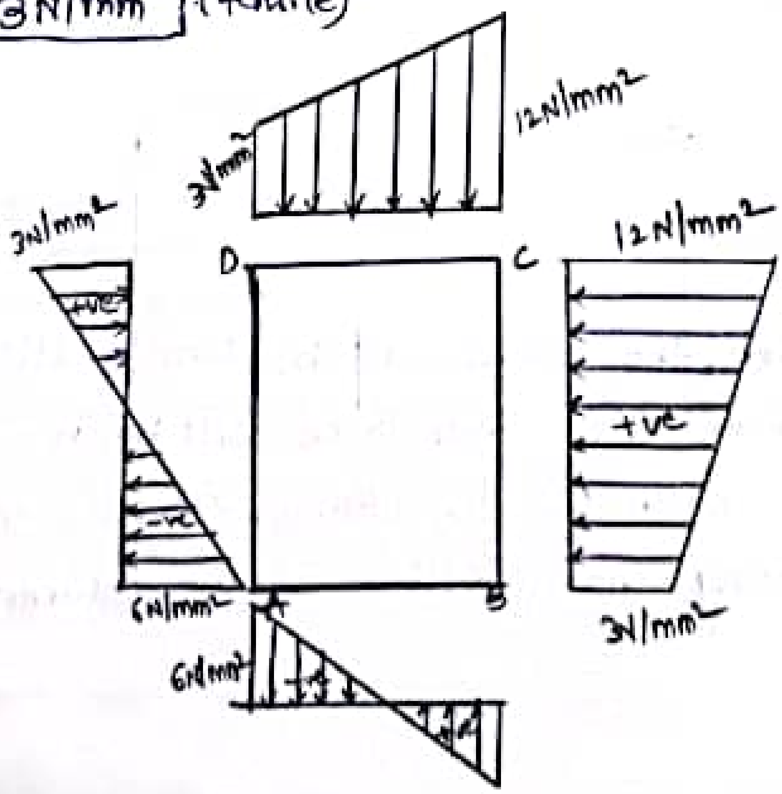
$\boxed{\sigma_C = 12 N/mm^2}$ (compressive)

Stress @ pt D:-

$x = -150$ mm, $y = 200$ mm

$$\sigma_D = \frac{-360\times10^3}{120000} + \frac{360\times10^3\times100\times200}{16\times10^8} - \frac{360\times10^3\times75\times150}{9\times10^8}.$$

$\boxed{\sigma_D = 3 N/mm^2}$ (tensile)



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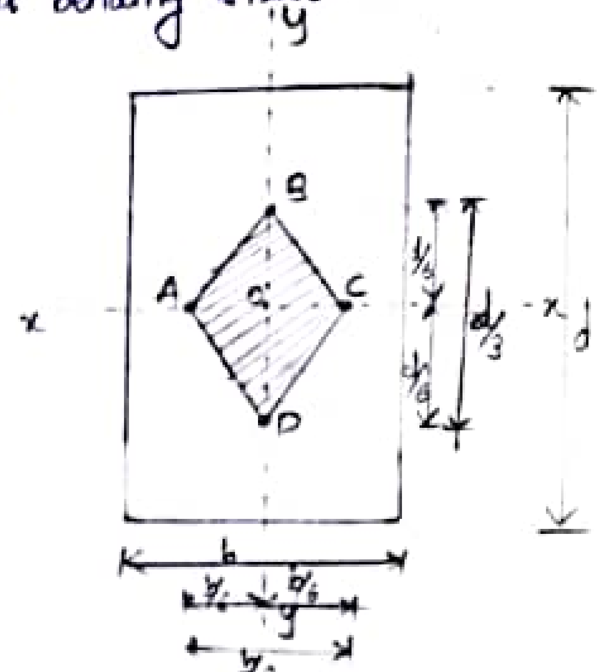
* Middle third Rule of a Rectangular section (Kernel of a section):-

The cement concrete columns are weak in tension. The load must be ~~equal~~ applied on this column in a such a way that there is a no tensile stress any where in the section. But any eccentric load acting on a column, it produces direct stresses as well as bending stresse. The resultant stress at any point in the section is the algebric sum of direct stress and bending stress.

Consider a rectangular section of width 'b' and depth 'd' as shown in the figure. Let this section is subjected to a load which is eccentric to y-y axis.



for a rectangular section

$$\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right]$$

If σ_{min} is -ve then, the stress will be tensile stress. But if, σ_{min} is +ve or '0'. then there will be no tensile stresses along the width of the column. Hence, for no, tensile stresses along the width of the column, then $\sigma_{min} \geq 0$.

$$\frac{P}{A}\left[1 - \frac{6e}{b}\right] \geq 0$$

$$1 - \frac{6e}{b} \geq 0$$

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$$1 \geq \frac{6e}{b}$$

$$\frac{b}{e} \geq 6$$

$$\frac{b}{6} \geq e$$

$$e \leq \frac{b}{6}$$

$\sigma_{min} \geq 0$

$$\frac{P}{A}\left[1 - \frac{6e}{d}\right] \geq 0$$

$$1 - \frac{6e}{d} \geq 0$$

$$1 \geq \frac{6e}{d}$$

$$e \leq \frac{d}{6}$$

$\therefore\ e \leq d/6$

The above results shows that the eccentricity 'e' must be less than equal to $b/6$

$\therefore$ the greatest eccentricity of load is $b/6$ from y-y axis.

Hence, if the load is applied at any distance less than $b/6$ from the axis on any side of y-y, the stresses are wholely compressive. Hence, the range with in which the load may be applied so as not to produce any tensile stress is with in the middle third of the base.

Similarly, if the load is eccentric to the axis x-x, the condition that tensile stress will not occur is when the eccentricity of the load with respect to the axis x-x, the condition that tensile stresses will not occur is when eccentricity of the load with respect to axis x-x does not exceed $d/6$. Hence, the range with in which the load may be applied is with in the middle third of the depth.

The tensile stresses will not occur when the load is applied any where, with in the rhombus ABCD whose diagonals are $AC = b/3$ & $BD = d/3$.

This figure ABCD with in which, the load may be applied any where so as not to produce tensile stress in any part of the rectangular section is called core or kernel of the section. Hence, the kernel of the section is the area with in which, the line of action of eccentric load 'p'. must cut the cross section if the stress is not to become in any part of the entire rectangular section.

Note:-

1) If direct stress σ_d is equal to bending stress σ_b then the tensile stresses will be zero.

2) If $\sigma_d > \sigma_b$ then stresses will be compressive through out the section.

3) If $\sigma_d < \sigma_b$ then stresses will be tensile through out the section.

4) Hence, for no tensile stresses, $\sigma_d \geq \sigma_b$

* Middle Quater Rule of a circular section :-

we know that

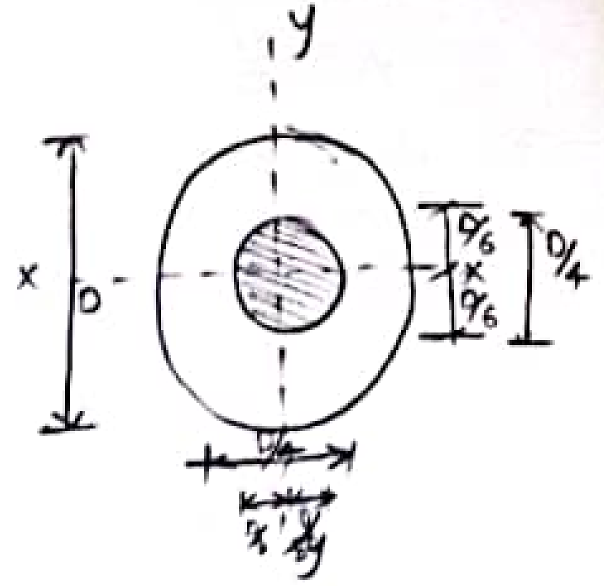
$\sigma_{min} = \sigma_d - \sigma_b$

But

$\sigma_d = \frac{P}{A}$

Now

$\sigma_b = \frac{My}{I}$



$$\sigma_b = \frac{Pe \times D/2}{\frac{\pi}{64} \times D^4} = \frac{PeD}{2} \times \frac{64}{\pi D^4}$$

$$\sigma_b = \frac{32Pe}{\pi D^3}.$$

Now,

$$\sigma_d = \frac{P}{A} = \frac{P}{\frac{\pi}{4}D^2} = \frac{4P}{\pi D^2}$$

Now,

$$\sigma_{min} = \sigma_d - \sigma_b$$

$$= \frac{4P}{\pi D^2} - \frac{32Pe}{\pi D^3}$$

$$= \frac{4P}{\pi D^2}\left[1 - \frac{8e}{D}\right]$$

for no tensile stresses

$$\sigma_{min} \geq 0$$

$$\frac{4P}{\pi D^2}\left[1 - \frac{8e}{D}\right] \geq 0$$

$$1 - \frac{8e}{D} \geq 0$$

$$1 \geq \frac{8e}{D}$$

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$$\frac{D}{8} \geq e$$

$$e \leq D/8$$

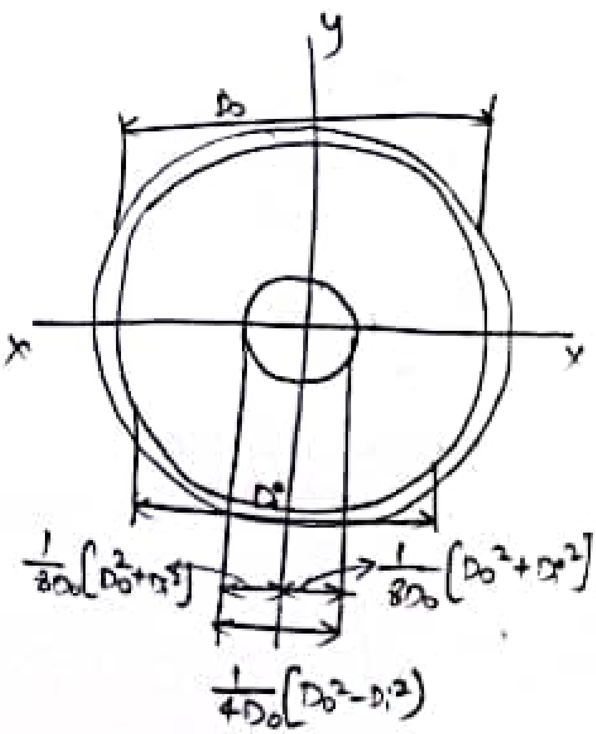
The above results shows that, the eccentricity 'e' must be less than or equal to D/8, it means that load can be essentric on any side of the centre of the circle by an amount equal to D/8. Thus, if the line of action of load is with in a circle of diameter equal to one fourth of main circle as shown in the figure then the stresses with will be compressive through out the circle section.

* Kernel of a hollow circular section (or) value of eccentricity for hollow circular section:-

we know that,

$$I = \frac{\pi}{64}\left[D_o^4 - D_i^4\right]$$

$$A = \frac{\pi}{4}\left[D_o^2 - D_i^2\right]$$



we know that

$$\sigma_{min} = \sigma_d - \sigma_b$$

But

$$\sigma_d = P/A$$

Now, we know that

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = \frac{M}{\left(\frac{I}{y}\right)}$$

Now, for no

But $Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}\left[D_o^4 - D_i^4\right]}{D_o/2}$

$$Z = \frac{\pi}{32D_o}\left[D_o^4 - D_i^4\right]$$

$$\sigma_b = \frac{M}{Z}$$

for, for no tensile stresses.

$$\sigma_d \geq \sigma_b$$

$$\frac{P}{A} \geq \frac{M}{Z}$$

$$\frac{P}{A} \geq \frac{Pe}{Z}$$

$$\frac{1}{A} \geq \frac{e}{Z}$$

$$\frac{Z}{A} \geq e$$

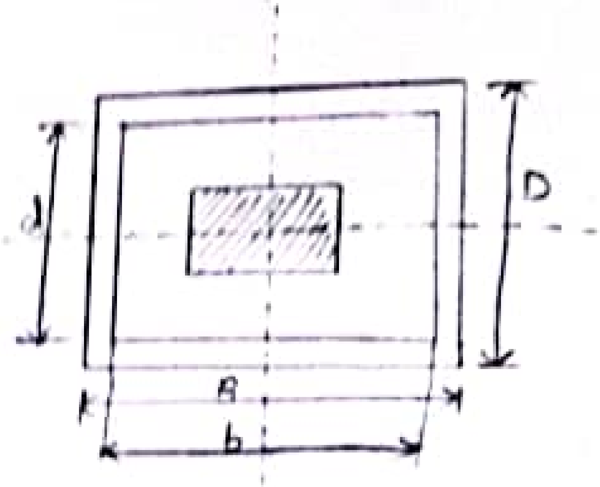
$$e \leq Z/A$$

$$e \leq \frac{\frac{\pi}{32D_o}\left[D_o^4 - D_i^4\right]}{\frac{\pi}{4}\left[D_o^2 - D_i^2\right]}$$

$$e \leq \frac{\frac{\pi}{32D_o}\left[D_o^2 + D_i^2\right]\left[D_o^2 - D_i^2\right]}{\frac{\pi}{4} \times \left[D_o^2 - D_i^2\right]}$$

$$e \leq \frac{1}{8D_o}\left[D_o^2 + D_i^2\right]$$

* Kernel of a hollow rectangular section (or) value of eccentricity for hollow rectangular section:-



We know that

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$A = BD - bd$$

$$I_{yy} = \frac{DB^3}{12} - \frac{db^3}{12}$$

Now $\sigma_d = \frac{P}{A}$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = \frac{M}{\left(\frac{I}{y}\right)}$$

Now, $Z_{xx} = \frac{I}{y_{max}} = \frac{I_{xx}}{y}$

$$= \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{D/2}$$

$$Z_{xx} = \frac{BD^3 - bd^3}{6D}$$

Now,

$$Z_{yy} = \frac{I_{yy}}{y} = \frac{\frac{DB^3}{12} - \frac{db^3}{12}}{B/2}$$

$$Z_{yy} = \frac{DB^3 - db^3}{6B}$$

Now, Along x-x axis

for, no tension stresses

$$\sigma_d \geq \sigma_b$$

$$\sigma_{bx} = \frac{M}{Z_{xx}} = \frac{P \times e}{\frac{BD^3 - bd^3}{6D}}$$

Now,

$$\sigma_d \geq \sigma_b$$

$$\frac{P}{A} \geq \frac{Pe}{Z_{xx}}$$

$$\frac{Z_{xx}}{A} \geq e$$

$$e_{xx} \leq \frac{Z_{xx}}{A}$$

$$e_{xx} \leq \frac{\frac{BD^3 - bd^3}{6D}}{(BD - bd)}$$

Now, Along y-y axis
for no tensile stresses

$$\sigma_d \geq \sigma_b$$

$$\frac{P}{A} \geq \frac{Pe_{yy}}{Z_{yy}}$$

$$e_{yy} \leq \frac{Z_{yy}}{A}$$

$$e_{yy} \leq \frac{\frac{DB^3 - db^3}{6B}}{(BD - bd)}$$

$$e_{yy} \leq \frac{DB^3 - db^3}{6B(BD - bd)}$$

* Draw the neat sketches of kernel of the following half cross-sections:

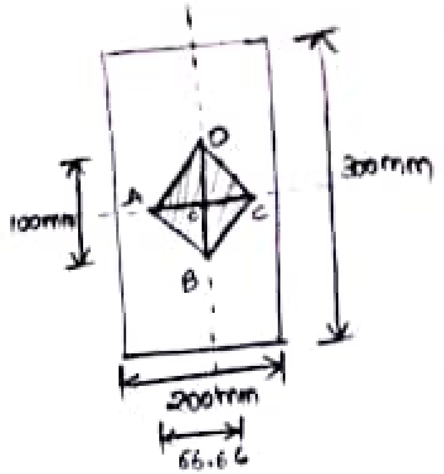
1) Rectangular section : 200 x 300mm.
2) Hollow circular cylinder with external diameter 300mm and thickness 50mm.
3) Square section with an area $400 cm^2$.

Soln:- Given.

$b = 200 mm$
$d = 300 mm.$

Now
$e_x \le \frac{d}{6}$

$e_x \le \frac{300}{6} = 50 mm$



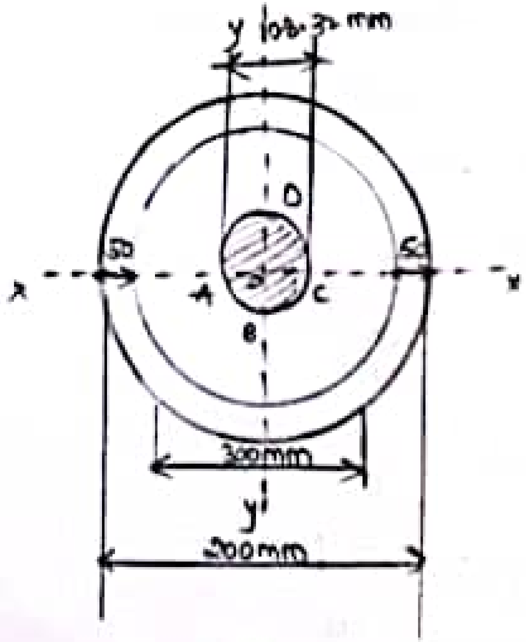
Now
$e_y \le \frac{b}{6}$

$e_y \le \frac{200}{6} = 33.33 mm$

Here,

$OC = OA = 33.33 mm$
$OB = OD = 50 mm$
Diagonals $AC = 66.66 mm$
$BD = 100 mm$

ii)



$D_o = 300mm$
$t = 50mm$
So, $D_i = D_o - 2t$
$= 300 - 2 \times 50$
$D_i = 200mm$

Here
$e \le \frac{1}{8 D_o}(D_o^2 + D_i^2)$

$$e \le \frac{1}{8\times 300}(300^2+200^2)$$

$e \le 54.16$ mm

Here,

$OA = OB = OC = OD = 54.16$ mm

Diameter $AC = BD = 108.32$ mm

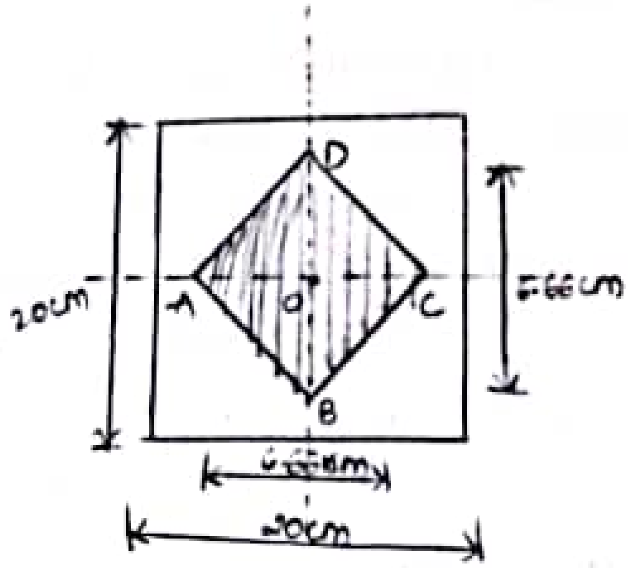
iii) Given:

$A = 400\,cm^2$

w.k.t Area of a square $= s^2$

$400 = s^2$

$s = 20$ cm



Here, $e_x = e_y \le \frac{s}{6}$

$\le \frac{20}{6} = 3.33$ cm

Here $OA = OB = OC = OD = 3.33$ cm

Diameter $AC = BD = 6.66$ cm.

• Draw the neat sketches of kernel of hollow Rectangular section of outer cross-section 300X200mm and inner cross-section 150X100mm

soln:- Given

$b = 150$ mm

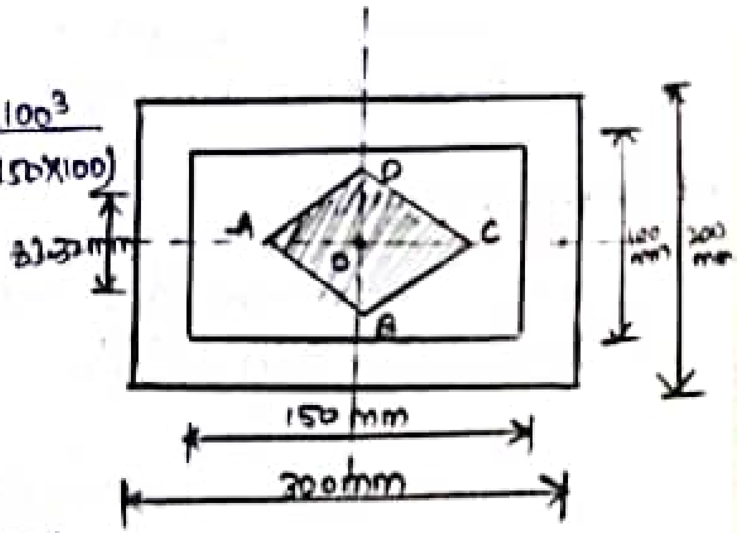
$d = 100$ mm

$$e_x \le \frac{d}{6}\,\frac{BD^3-bd^3}{6D(BD-bd)} = \frac{300\times200^3-150\times100^3}{6\times200(300\times200-150\times100)}$$

$e_x \le$ $\Rightarrow$ 41.66 mm

Now $e_y \le \frac{DB^3-db^3}{6B(DB-bd)}$

$$e_y \le \quad = \frac{200\times300^3-100\times150^3}{6\times300(300\times200-100\times150)}$$



$e_{yy} \le 62.5$ mm.

Now, $OA = OC = 62.5$ mm.

Here $OB = OD = 41.66$ mm

Diagonals $= AC = 125$ mm

$BD = 83.32$ mm

* DAMS :-

A large quantity of water is required for irrigation & power generation throughout the year. A dam is constructed to store water. The water stored in a dam exerts the pressure force on the face of the dam. In contact with water. Here we shall study different type of dams, stress across the section of dam, stability of dam and minimum bottom width required for a dam section.

* Types of dams :-

There are many types of dams but the following are the important.

1) Rectangular dam

2) Trapezoidal dam.

Trapezoidal dam has compared with rectangular dam, it is more economical and easier to construct. Hence these days trapezoidal dams are mostly constructed.

* Rectangular dams :-

Consider a rectangular section of dam which is stored with water at a height 'h'.

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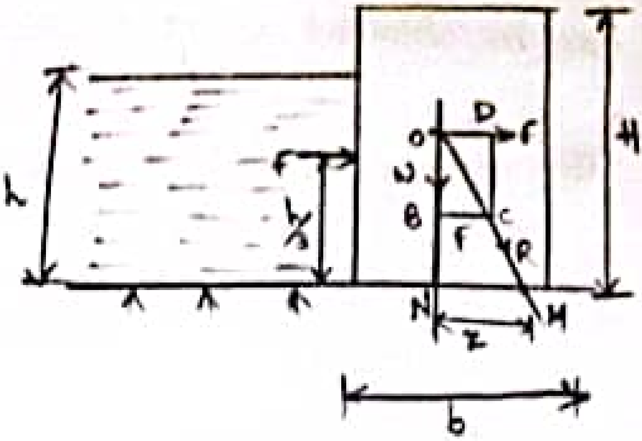
Now L = length of dam.
consider $L = 1m$.



w_0 = weight density of dam.
w = weight density of water.
W = weight of the dam.
b = width of the dam.
H = Height of the dam.
h = height of the water.

The forces acting on the dam are
i) The force F due to water in contact with side of the dam.
the force is given by $F = \rho g A\bar{h}$

$$F = wA\bar{h}$$
$$= w \times h \times L \times \bar{h}$$
$$= w \times h \times 1 \times \frac{h}{2}$$
$$\boxed{F = \frac{wh^2}{2}}$$

The force F will be acting horizontally at a height of $h/3$ above the base as shown in the figure.

ii) ~~The weight of~~ the dam W and is given as
weight force, W
$$W = \text{weight density of dam} \times \text{volume}$$
$$= w_0 \times A \times L$$
$$= w_0 \times bH \times 1$$
$$\boxed{W = w_0 bH}$$

The weight W acting downwards through the centre of gravity of dam. as shown in the figure

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iii) There are two forces acting along the dam, the resultant can be obtained as $\boxed{R = \sqrt{F^2+W^2}}$

The angle made by the resultant is

$$\tan\theta = \frac{BC}{BO}$$

$$\tan\theta = \frac{F}{W}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{F}{W}\right)}$$

iv) Horizontal distance b/w line of action W. and point through which resultant cuts the base.

For the figure the distance x is obtained from Δ^{le} NMO, Δ^{le} BCO

Both are similar Δles.

$$\frac{NM}{NO} = \frac{BC}{BO}$$

$$\frac{x}{h/3} = \frac{F}{W}$$

$$\frac{3x}{h} = \frac{F}{W}$$

$$\boxed{x = \frac{Fh}{3W}}$$

1) A masonry dam rectangle section 20m height and 10m wide has water upto a height of 16m on its one side. Determine

i) pressure force due to water on one 1 1m length of dam.

ii) position of center of pressure.

iii) The ~~moment~~ point at which resultant cuts the base.

Take weight density of Masonry $= 19.62\ kN/m^3$ and water $= 9810\ N/m^3$.

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Sol^n Given.

$H = 20m$

$b = 10m$

$h = 16m.$

$\lambda = 1$

$w_0 = \rho_{masonry} = 19.62 \ kN/m^3 = 19.62 \times 10^3 \ N/m^3.$

$w = \rho_{water} = 1000 \ N/m^3.$

i) w.k.t

$f = \frac{wh^2}{2}$

$f = \frac{\rho g h^2}{2} = \frac{1000 \times 9.81 \times 16^2}{2} = 1255680 \ N.$

$\boxed{f = 1255680 \ N}$

ii) position of centre of pressure

$= \frac{h}{3}$

$= \frac{16}{3} = 5.33$ m from base

iii) w.k.t

$R = \sqrt{f^2 + W^2}$

But

$W = w_0 bH$

$= \rho_{masonry} \times g \times 10 \times 20$

$= 19.62 \times 10^3 \times \cancel{9.81} \times 10 \times 20$

$W = \cancel{3849444 \ N}.$ $3924000 N$

Now, $R = \sqrt{1255680^2 + \cancel{3849444^2} (3924000)^2}$

~~R = 4049067.46 N~~

$R = 4120031.2 \ N$

w.k.t

$$x = \frac{Fh}{3W} = \frac{1255680 \times 16}{3 \times \cancel{3449944}(3929000)}$$

$\boxed{x = 1.70\,m}$ from centroid of given dam section.

2) A Masonry dam of rectangular cross-section 10m height and 5m wide has water upto top on its one side. If the weight density of Masonry is 21.582 KN/m³, find

i) pressure force due to water per m length of dam.

ii) The resultant force and the point at which it cuts the base of the dam.

Soln:- given

$H = 10m = h.$

$b = 5m$

$w = \rho g = 9810\ N/m^3.$

$L = 1m$

W_0 ~~Smasonry~~ $= 21.582\ KN/m^3 = 21.582 \times 10^3\ N/m^3$

$\rho_{water} = 1000\ N/m^3.$

i) w.k.t

$$F = \frac{wh^2}{2}$$

$$= \frac{9810 \times 10^2}{2}$$

$\boxed{F = 490500}$

ii)

$$x = \frac{Fh}{3W}$$

$$x = \frac{490500 \times 10}{3 \times W} \quad \text{—①}$$

Now

$W = w_0 bH$

$= \rho_{masonry} \times g \times b \times H$

$= 21.582 \times \cancel{9.81} \times 5 \times 10 \times 10^3$

$\boxed{\cancel{W = 1058597.1 N}}$ $\boxed{1079100}$ N

Now sub in ①

$x = \dfrac{490500 \times 10}{3 \times 1058597.1}$

$\boxed{\cancel{x = 0.154}}$ m $\boxed{x = 1.51 m}$

$R = \sqrt{F^2 + W^2}$

$= \sqrt{490500^2 + 1058597.1^2}$

$\boxed{\cancel{R = 1059732.3.54 N}}$ $\boxed{R = 1185346.81 N}$

$x = \overset{1.51\,m}{\cancel{0.15 m}}$ from centroid of the given ~~se~~ dam section.

* Stress across a section of a rectangular dam.

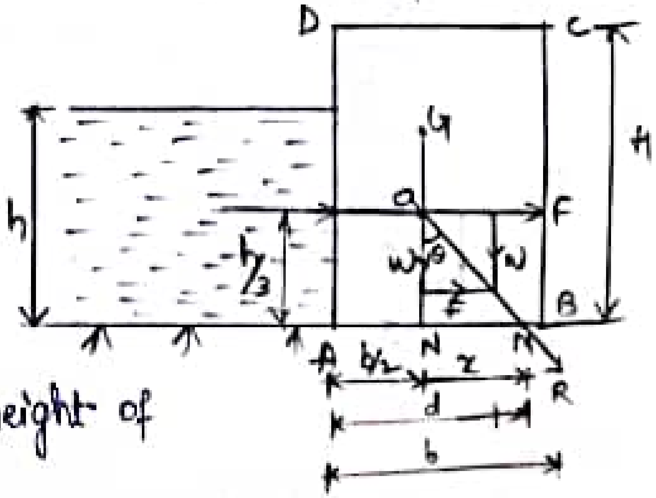

figure shows a rectangular dam of height H and width 'b'. The dam is having water upto the depth of 'h'. The forces acting on the dam are

i) pressure force, $F = \dfrac{wh^2}{2}$ at a height of $\dfrac{h}{3}$ from base.

ii) Self weight, $W = w_0 bH$ at centre of gravity of dam. (CG)

iii) The Resultant force $R = \sqrt{F^2 + W^2}$.

· The Resultant cuts the base at a distance $x = \dfrac{Fh}{3W}$ from the

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Centroid of dam.

From figure $d = \frac{b}{2} + x$.

$$d = \frac{b}{2} + \frac{Fh}{3W}$$

The resultant 'R' meets the base at M. The resultant force 'R' acting at 'M' can be resolved into vertical component 'W' and Horizontal component 'F' as shown in the figure.

The vertical component W acting at point M on the base of the dam is eccentric load as it is not acting at the middle of the base.

The Eccentric load produces direct stress and Bending stress.

eccentricity $e = x = d - b/2$

We know that $\sigma_{max} = \sigma_b + \sigma_d$

where $\sigma_d = \frac{W}{A}$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = \frac{We \times \frac{b}{2}}{\frac{b^3}{12}} \quad \left[\because I = \frac{db^3}{12} \text{ \& } d = 1\right]$$

$$\sigma_b = \frac{Web}{2} \times \frac{12}{b^3}$$

$$\sigma_b = \frac{6We}{b^2}$$

Now

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$= \frac{W}{A} + \frac{6We}{b^2}$$

$$= \frac{W}{1 \times b} + \frac{6We}{b^2}$$

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$$\boxed{\sigma_{max} = \frac{W}{b}\left[1+\frac{6e}{b}\right]}$$

$$\sigma_{min} = \sigma_d - \sigma_b$$

$$= \frac{W}{A} - \frac{6We}{b^2}$$

$$= \frac{W}{1\times b} - \frac{6We}{b^2}$$

$$\boxed{\sigma_{min} = \frac{W}{b}\left[1-\frac{6e}{b}\right]}$$

∴ If the σ_{min} is $-$ve the stress is tensile.
σ_{min} is $+$ve then the stress is compressive.

1) A masnory dam of rectangle section 20m height and 10m has water upto a height of 16m. find the maximum and minimum intensites of at base of the dam. The wt density of Masnory $= 19.62$ kN/m³

Soln:- Given

$H = 20m$

$b = 10m$

$h = 16m$

$w_0 = 19.62$ kN/m³

$= 19.62\times10^3$ N/m³

w.k.t

$$\sigma_{max} = \frac{W}{b}\left[1+\frac{6e}{b}\right]$$

But $W = w_0 bH = 19.62\times10^3\times10\times20$

$W = 3924000$ N

Now,

$$e = \frac{Fh}{3W} = x$$

But $F = \frac{wh^2}{2} = \frac{9810\times16^2}{2} = 1255680$ N

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Now,

$$e = \frac{1255680 \times 16}{3 \times 3924000}$$

$$e = 1.70\,m$$

Now.

$$\sigma_{max} = \frac{3924000}{10}\left[1 + \frac{6 \times 1.70}{10}\right]$$

$$\boxed{\sigma_{max} = 792643\ N/m^2}$$ (compressive)

Now

$$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{3924000}{10}\left[1 - \frac{6 \times 1.70}{10}\right]$$

$$\boxed{\sigma_{min} = -78.48\ N/m^2}$$ (Tensile)

* Trapezoidal dam having water face vertical :-

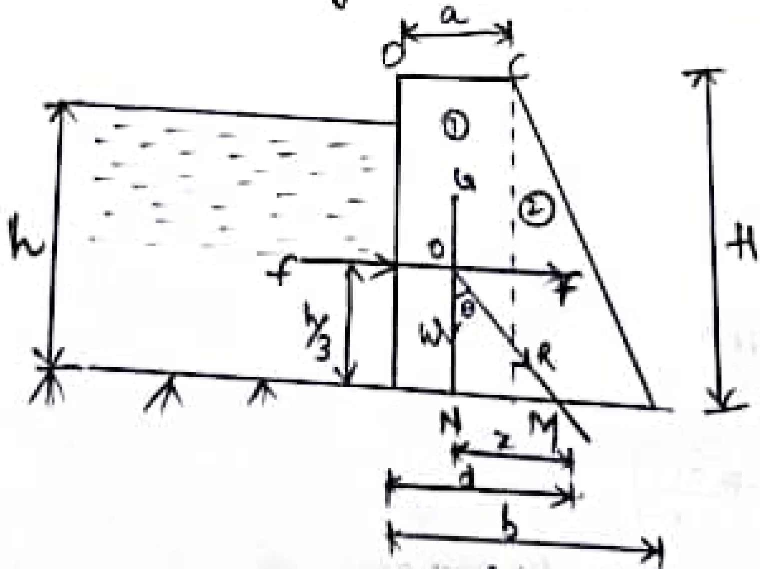

figure shows a trapezoidal dam having water face vertical. consider 1m length of the dam.

let H = Height of the dam

h = Height of the water.

a = top width of the dam.
b = bottom width of the dam.
w_0 = weight density of Masnory.
w = weight density of water.
F = pressure force exerted by water.
W = weight of dam per metre length.

Now, Forces acting on the dam are i) water force acting on the dam & F.

i) w.k.t

$$F = \rho g A \bar{h}$$

$$= wA\bar{h}$$

$$= w \times h \times 1 \times h/2$$

$$\boxed{F = \frac{wh^2}{2}}$$

ii) weight force acting on the dam
w.k.t

W = wt. density of dam × vol.

$$= w_0 A \times L$$

$$= w_0 \times \frac{1}{2} H(a+b) \times 1$$

$$\boxed{W = \frac{w_0 H(a+b)}{2}}$$

iii) Resultant

$$R = \sqrt{F^2 + W^2}$$

$$\tan\theta = \frac{F}{W}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{F}{W}\right)}$$

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Now, $x = \frac{Fh}{3W}$

$e = d - b/2$

But $d = AN + NM$

$d = AN + x$

Now $A\bar{x} = A_1\bar{x_1} + A_2\bar{x_2}$

$$\frac{1}{2}H(a+b) \times AN = aH \times \frac{a}{2} + \frac{1}{2} \times (b-a) \times H \times \left(\frac{b-a}{3} + a\right)$$

(or)

$$AN = \frac{a^2+ab+b^2}{3(a+b)}$$

Now,

$$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]$$

$$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$$

1) A Trapezoidal Masnory dam is of 18 m height, the dam is having water upto a depth of 15m on its vertical side. The top and bottom width of the dam are 4m and 8m respectively. The weight density of masnory is given as 19.62 kN/m³. Determine i) The resultant force on the dam. per meter length. ii) the point where the resultant cuts the base. iii) The maximum and minimum intensities of stress, at the base.

Soln:- $H = 18m$

$h = 15m$

$a = 4m$

$b = 8m$

$w_0 = 19.62\ kN/m^3$
$= 19620\ N/m^3$.

i) w.k.t

$$F = \frac{wh^2}{2}$$

$$= \frac{9810 \times 15^2}{2}$$ · [wt density of water $w = 9810$]

$$F = 1103625\ N.$$

Now,

$$W = \frac{w_0 H(a+b)}{2}$$

$$= \frac{19620 \times 18 \times (4+8)}{2}$$

$$W = 2118960\ N.$$

Now, w.k.t

$$R = \sqrt{F^2 + W^2}$$

$$= \sqrt{1103625^2 + 2118960^2}$$

$$R = 2389137.84\ N.$$

ii) w.k.t

$$x = \frac{Fh}{3W}$$

$$x = \frac{1103625 \times 15}{3 \times 2118960}$$

$$x = 2.60\ m$$

Now, $AN = \frac{a^2+ab+b^2}{3(a+b)}$

$$= \frac{4^2 + 4 \times 8 + 8^2}{3(4+8)}$$

$AN = 3.11\ m$

Now $d = AN + x$

$= 3.11 + 2.60$

$d = 5.71 m$ from vertical face of the water.

iii) W.k.t

$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]$

Act,

$e = d - b/2$

$= 5.71 - \frac{8}{2}$

$e = 1.71 m$

Now,

$$\sigma_{max} = \frac{2118960}{8}\left[1 + \frac{6 \times 1.71}{8}\right]$$

$\sigma_{max} = 604565.77\ N/m^2$.

Now

$$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{2118960}{8}\left[1 - \frac{6 \times 1.71}{8}\right]$$

$\sigma_{min} = -74825.97\ N/m^2$ (tensile)

2) A Trapezoidal Masonry dam of 4m height, 1m wide at its top and 3m width at its bottom. Retains water on its vertical face. Determine the maximum and minimum stresses at the base. i) When the reservoir is full. ii) when the reservoir is empty.
Take the weight density of Masonry is 19.62 kN/m³.

Soln: Given data.

$H = 4m$

$a = 1m$

$b = 3m$

$w_0 = 19.62 \ kN/m^3$

$= 19620 \ N/m^3.$

i) when reserviour is full :-

$H = h = 4 \ m.$

$$F = \frac{wh^2}{2}$$

$$= \frac{9810 \times (4)^2}{2}$$

$$F = 78480 \ N$$

Now,

$$W = \frac{w_0 H (a+b)}{2}$$

$$= \frac{19620 \times 4 \times (1+3)}{2}$$

$$W = 156960 \ N$$

w.k.t

$$x = \frac{Fh}{3W} = \frac{78480 \times 4}{3 \times 156960}$$

$$x = 0.66 \ m$$

Now,

$$AN = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$= \frac{1^2 + 1 \times 3 + 3^2}{3(1+3)}$$

$$AN = 1.08 \ m.$$

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Now,

$$d = AN + x$$
$$= 1.08 + 0.66.$$

$d = 1.74\,m$ from vertical face of the water

iii) w.k.t

$$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]$$

But

$$e = d - \frac{b}{2}$$
$$= 1.74 - \frac{3}{2}$$
$$e = 0.24\,m.$$

Now.

$$\sigma_{max} = \frac{156960}{3}\left[1 + \frac{6 \times 0.24}{3}\right]$$
$$\sigma_{max} = 77433.6\ N/m^2$$

Now, $\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$

$$= \frac{156960}{3}\left[1 - \frac{6 \times 0.24}{3}\right]$$
$$\sigma_{min} = 27206.4\ N/m^2.$$

ii) when reservoir is empty :- when reservoir is empty there is no pressure force.

$F = 0$

$\therefore F = 0$

Now, $W = \frac{W_0 H(a+b)}{2}$

$$= 19620 \times 4\,(1+3)$$
$$W = 156960\ N$$

Now, $x=0$.

Now, $AN = \frac{a^2+ab+b^2}{3(a+b)}$

$AN = 1.08\ m$

$d = AN + x$

$= 1.08 + 0$

$d = 1.08\ m$ from vertical face of water

w.k.t.

$\sigma_{max} = \frac{W}{b}\left[1+\frac{6e}{b}\right]$

Now

$e = d - \frac{b}{2}$

$= 1.08 - \frac{3}{2}$

$e = -0.42\ m$ [-ve sign only indicates that stress at A will be more than stress at B]. $\therefore e = 0.42\ m$.

$\sigma_{max} = \frac{156960}{3}\left[1+\frac{6(0.42)}{3}\right]$

$\sigma_{max} = 96268.8\ N/m^2$

Now

$\sigma_{min} = \frac{W}{b}\left[1-\frac{6e}{b}\right]$

$= \frac{156960}{3}\left[1-\frac{6(0.42)}{3}\right]$

$\sigma_{min} = 8371.2\ N/m^2$

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* **Stability of a dam:-**

→ A dam should be stable under all conditions. But the dam may fail.

1. by sliding on the soil on which it rests.
2. By overturning.
3. Due to tensile stresses developed, and.
4. due to excessive compressive stresses.

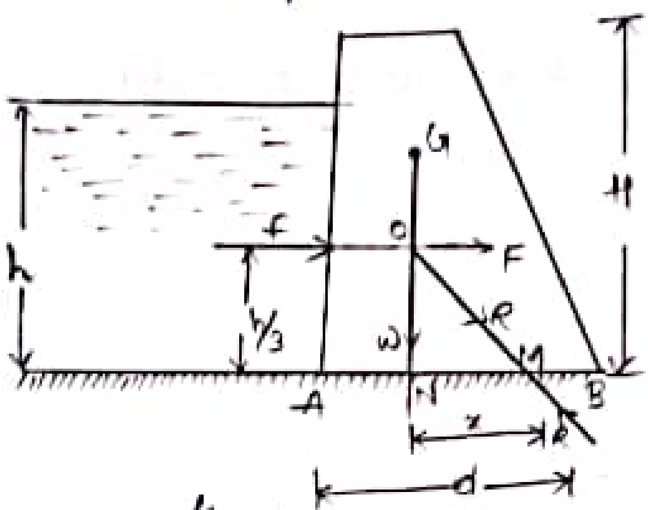

fig:-1

i) **condition to prevent the sliding of the dam:-** fig. 1 shows a dam of trapezoidal section of height H and having water upto a depth of h. the forces acting on the dam are:

i) force due to water pressure F acting horizontally at a height of $\frac{h}{3}$ above the base.

ii) Weight of the dam W acting vertically downwards through the C.G of the dam.

The Resultant R of the forces F and W is passing through the point M. The dam will be in equilibrium if a force R^* equal to R is applied at the point M. the dam ~~will be in~~ R in the opposite direction of R. Here R^* is the reaction of the dam. The Reaction R^* can be resolved into the two components. The vertical component of R^* will be equal to W

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whereas the horizontal component will be equal to frictional force at the base of the dam.

let μ = coefficient of friction between the base of the dam and the soil.

Then Maximum force of friction is given by,

$$F_{max} = \mu \times W.$$

If the force of friction i.e F_{max} is more than the force due to water pressure (i.e F), the dam will be safe against sliding.

ii) Condition to prevent the overturning of the dam:-

If the Resultant R of the weight W of the dam and the horizontal F due to water pressure, strikes the base within its width i.e the point M lies within the base AB of fig.1, there will be no overturning of the dam. This is proved as.

For the dam shown in fig.1 taking moments about M,

Moment due to horizontal force F about point M.

$$= F \times \frac{h}{3} \quad \text{—(1)}$$

Moment due to weight W about point M.

$$= W \times x. \quad \text{—(2)}$$

∴ The moment, due to horizontal force F, tends to overturn the dam about the point B; whereas the moment due to weight W tends to restore the dam. If the moment due to weight W is more than the moment due to force F, there will be no overturning of the dam. for the equillibrium of the dam, the two moments should be equal.

$$\therefore F \times \frac{h}{3} = W \times x. \quad \text{—(3)}$$

Since overturning can take place about point B, hence restoring moment about the point B

$= W \times NB$

But overturning moment due to force F about point B.

$= F \times \frac{h}{3} = W \times x$

There will be no overturning about point B, if restoring moment about B is more than the overturning moment about B, i.e

$\cancel{W} \times NB > \cancel{W} \times x.$

$NB > x.$

$> NM \qquad (\because x = NM)$

This means that there will be no overturning of the dam if point M lies between N and B or between A and B.

iii) Condition to Avoid Tension in the Masonry of the dam at its Base :-

The Masonry of the dam is weak in tension and hence the tension in the Masonry of the dam should be avoided.

The maximum and minimum stresses across the base of the dam are given by eqn ① & ②.

The maximum stresses is always compressive but the minimum stress given by eqn ② will be tension if the term $\left(1 - \frac{6e}{b}\right)$ is negative. In limiting case, there will be no tensile stress at the base of dam.

if $1 - \frac{6e}{b} \geq 0$

$b - 6e \geq 0$ (or) $b \geq 6 \cdot e$

$6e \leq b$ (or) $e \leq \frac{b}{6}$ — ④

e = Eccentricity & b = Base width of dam.

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This means that the base section. Hence the resultant must lie within the middle third of the base width, in order to avoid tension.

If d^* = Maximum distance between A and the point through which resultant force R meets the base.

Then $e = d^* - b/2$ —①

But to avoid tension at the base of the dam, Maximum value of eccentricity.

$e \le b/6$ —②

from equation ① & ②.

$d^* - b/2 \le b/6$

$\therefore \; d^* \le b/6 + b/2 \le \frac{b+3b}{6} \le \frac{4b}{6} \le \frac{2}{3}b$

Hence if the maximum distance between A and the point through which resultant force R meets the base (i.e. distance d) is equal to or less than two third of the base width, there will be no tension at the base of dam.

iv) Condition to avoid the Excessive compressive stresses at the Base of the dam:-

The maximum and minimum stresses across the base of the dam are given by eqn ① & eqn ②.

The condition to avoid the excessive compressive stresses in the masonry of the dam is that the p_{max} i.e. maximum stress in the masonry should be less than the permissible stress in the masonry.

1) A Trapezoidal Masonary having 4m top width, 8m bottom width & 12m high is retaining water up to a height of 10m as shown in the figure. The density of Masnory is 2000kg/m³ and coefficient of friction between the dam & soil is 0.55. The allowable compressive stress is 343350 kN/m². Check the stability of dam.

Soln:- Given data.

$a = 4m$

$b = 8m$

$H = 12m$

$h = 10m.$

$\rho = 2000\ kg/m^3$

$\sigma_{allowable} = 343350\ N/m^2$

$\mu = 0.55$

Now, $w_0 = \rho g$

$= 2000 \times 9.81$

$w_0 = 19620\ N/m^3.$

Now, i) check for sliding.

if $F_{max} > F$ then there will be no sliding.

Now, $F_{max} = \mu W$

But

$$W = \frac{w_0 (H)(a+b)}{2}$$

$$= \frac{19620 \times 12 \times (4+8)}{2}$$

$W = 1412640\ N.$

Now, $F_{max} = 0.55 \times 1412640$

$F_{max} = 776952$ N.

Now,

$$F = \frac{wh^2}{2}$$

$$= \frac{9810 \times (10)^2}{2}$$

$$F = 490500 \text{ N}$$

Now, $F_{max} > F$.

so there will be no sliding.

ii) check for overturning:-

Now

$$x = \frac{Fh}{3W}$$

$$= \frac{490500 \times 10}{3 \times 1412640}$$

$$x = 1.15 \text{ m}.$$

Now,

$$AN = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$= \frac{4^2 + 4 \times 8 + 8^2}{3(4+8)}$$

$$AN = 3.11 \text{ m}$$

Now, $d = AN + x$

$$= 3.11 + 1.15$$

$$d = 4.26 \text{ m}$$

As the resultant is cutting the base @ a dist 4.26 m from vertical force of the water which is ~~#~~ less than base width b = 8m of the dam

∴ The dam is safe against overturning.

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iii) Check for tensile stresses:-

Here $d < \frac{2}{3}b$

Now $d = 4.26\ m$

$\frac{2}{3}b = \frac{2}{3} \times 8 = 5.33\ m$

As dist d is less than $\frac{2}{3}b$.

i.e $4.26 \leq 5.33\ m$

tensile stresses will not be developed in the dam.

iv) Check for excessive compressive stress:-

Here $\sigma_{max} < \sigma_{allowable}$

Now

$$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]$$

But

$$e = d - b/2 = 4.26 - \frac{8}{2}$$

$$e = 0.26\ m$$

$$\sigma_{max} = \frac{1412640}{8}\left[1 + \frac{6 \times 0.26}{8}\right]$$

$$\sigma_{max} = 211013.1\ N/m^2$$

Now,

$\sigma_{allowable} = 343350\ N/m^2$.

As $\sigma_{max} < \sigma_{allowable}$ so

the compressive stresses which are formed are within the limit

So, the dam is safe.

∴ As all the 4 conditions are satisfied, the dam is stable.

2)-A Trapezoidal Masonry dam having top width 1m and height 8m is retaining water upto a height of 7.5m. The water face of the dam is vertical, the density of Masonry is 2240 kg/m³ and coefficient of friction b/w dam and soil is 0.6. Find the minimum bottom width of the dam required.

Soln- Given data.

$a = 1m$

$H = 8m$

$h = 7.5m$

$\rho = 2240\ kg/m^3$

$\mu = 0.6.$

Now $w_0 = \rho \times g.$

$= 2240 \times 9.81$

$w_0 = 21974.4\ N/m^3.$

Now, i) Check for sliding.

if $F_{max} > f$ there will be no sliding.

$F_{max} = \mu \times w$

$f_{max} = 0.6 \times W$

$$W = \frac{w_0 H(a+b)}{2}$$

$$W = \frac{21974.4 \times 8\ (a+b)}{2} \Rightarrow 87897.6\ (1+b)$$

Now.

$$F = \frac{wh^2}{2} \Rightarrow \frac{9810 \times (7.5)^2}{2}$$

$f = 275906.25\ N$

Now check for overturning.

$$x = \frac{Fh}{3W}$$

$$x = \frac{275906.25 \times 7.5}{3 \times 87897.6(1+b)}$$

$$x = \frac{7.84}{1+b}$$

$$AN = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$AN = \frac{1 + b + b^2}{3(1+b)}$$

$d = AN + x$.

$$= \frac{1+b+b^2}{3(1+b)} + \frac{7.84}{1+b}$$

$$d = \frac{1+b+b^2 + 3 \times 7.84}{3(1+b)}$$

$$= \frac{24.52 + b + b^2}{3(1+b)}$$

$$d = \frac{b^2 + b + 24.52}{3(1+b)}$$

But w.k.t $d \le \frac{2}{3} b$

for limiting condition.

$$d = \frac{2}{3} b$$

$$\frac{b^2 + b + 24.52}{\cancel{3}(1+b)} = \frac{2}{\cancel{3}} b$$

$$\frac{b^2 + b + 24.52}{1+b} = 2b$$

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$b^2 + b + 24.52 = 2b(1+b)$

$b^2 + b + 24.52 = 2b + 2b^2$

$b^2 - b + 2b - 24.52 = 0$

$b^2 + b - 24.52 = 0$

$\boxed{b = 4.47}$ — ①

Now $f_{max} > F$

$\mu W > \frac{wh^2}{2}$

$0.6(9897.6(1+b)) > \cancel{27497.6(2)} \cdot \frac{275906.25 \times (7.5)^2}{2}$

$b > \frac{275906.25}{0.6 \times 9879.6 (2.7)} - 1$

$\boxed{b > 4.27\,m}$ — ②.

Now, bottom width b of dam = greatest of two values of ① & ②

$b = \underline{4.47\,m}$

- Retaining Walls:-

The walls which are used for Retaining the soil are known as Retaining walls. The earth or soil Retained by Retaining wall exerts pressure on the Retaining wall in the same way as water exerts pressure on the dam.

A number of theories have being evolved to determine the pressure exerted by the soil on the retaining wall. one of the theories is Rankines theory of earth pressure before discussing Rankines theory let us define the angle of Repose.

* **Angle of Repose:-**

It is defined as maximum inclination of plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only. The earth particles lack in cohesion and have a definite angle of Repose. And the angle of Repose is equal to angle of friction.

* **Rankines theory of earth pressure:-**

Rankines theory of earth pressure is used to determine the pressure exerted by the earth or soil on the retaining wall. This theory is based on the following assumptions.

1) The earth or soil retained by a retaining wall is cohesionless
2) Frictional resistance b/w the retaining wall and the retained material is neglected.
3) The failure of the retained material takes place along a plane known as rupture plane.

The pressure exerted by the soil on the retaining wall is given by the formulae.

$$\boxed{p = \frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]}$$

where w = weight density of soil
h = Height of wall
ϕ = angle of Repose.

The ~~force~~ pressure force p is acting in the horizontal direction on the retaining wall at a distance of $h/3$ above the base.

* **Pressure intensity at the bottom:-**

If we assume a linear variation of pressure

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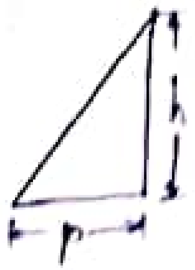
Intensity is varying from 0 at the top to the maximum value p at the bottom. then we have

$$P = \frac{ph}{2}$$

$$\frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right] = \frac{ph}{2}$$



$\because P = \frac{p \times h}{2}$

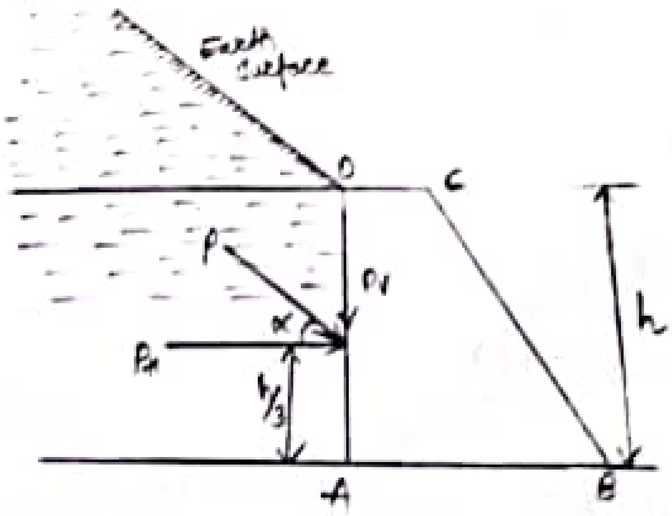
$$\boxed{p = wh\left[\frac{1-\sin\phi}{1+\sin\phi}\right]}$$

* **Surcharged Retaining Wall** :-

figure shows a retaining wall of height 'h' and retaining earth which is surcharged at an angle α with the horizontal. Then the total earth pressure exerted on the retaining wall is given by

$$\boxed{p^* = \frac{wh^2}{2}\cos\alpha\,\frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha + \cos^2\phi}}}$$

$\because$ where α = angle of surcharge



where,

$P_H = p\cos\alpha$

$P_V = p\sin\alpha.$

Q.) A masonry retaining wall of trapezoidal section is 6m hight and retains earth which is level upto the top. The width at the top is 1m. and the exposed face is vertical. find the minimum width of the wall at the bottom in order the tension may not be induced at the base. The density of Masonary and earth is 2300 and 1600 kg/m³ respectively. The angle of repose of the soil is 30°.

Soln:- Given:

$h = 6m$

$a = 1m$

$\rho_0 = 2300\ kg/m^3$

$\rho = 1600\ kg/m^3$

$\phi = 30°$

w.k.t

$$w = \rho g = 1600 \times 9.81 = 15696\ N/m^3.$$

$$w_0 = \rho_0 g = 2300 \times 9.81 = 22563\ N/m^3.$$

Now,

$$P = \frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]$$

$$= \frac{15696 \times 6^2}{2}\left[\frac{1-\sin 30°}{1+\sin 30°}\right]$$

$$P = 94176\ N.$$

w.k.t

$$W = \frac{w_0 h(a+b)}{2}$$

$$= \frac{22563 \times 6(1+b)}{2}$$

$$W = 67689(1+b)\ N$$

Now, $d = AN + x$

But $AN = \dfrac{a^2 + ab + b^2}{3(a+b)}$

$$AN = \frac{1^2 + 1 \times b + b^2}{3(1+b)}$$

$$AN = \frac{b^2+b+1}{3(1+b)}$$

Now, $x = \frac{Ph}{3W}$

$$x = \frac{94176 \times 6}{3 \times 67679(1+b)}$$

$$x = \frac{2.73}{1+b}$$

Now, $d = AN + x$

$$= \frac{b^2+b+1}{3(1+b)} + \frac{2.73}{1+b}$$

$$d = \frac{b^2+b+1+3\times2.73}{3(1+b)}$$

$$d = \frac{b^2+b+9.34}{3(1+b)}$$

for No tension to occur in the dam then $d \le \frac{2}{3}b$

at limiting condition,

$$d = \frac{2}{3}b.$$

$$\frac{b^2+b+9.34}{\cancel{3}(1+b)} = \frac{2}{\cancel{3}}b$$

$$b^2+b+9.34 = 2b+2b^2$$

$$b^2+b-9.34 = 0$$

$$\boxed{b = 2.59}\ m.$$

2) A Masonry retaining wall of trapezoidal section is of 10m height and retains earth which is level upto the top. The width at the top is 2m and at the bottom is 8m and exposed face is vertical. Find the maximum and minimum intensities of

the stress at the base. Take density of earth = 1600 kg/m³. Density of Masonry = 2400 kg/m³. Angle of repose = 30°.

Soln:- Given data:-

$a = 2m$

$b = 8m$

$h = 10m$

$\rho = 1600 kg/m^3$

$\rho_0 = 2400 kg/m^3$

$\phi = 30°$

We know that

$w = \rho g = 1600 \times 9.81 = 15696 \, N/m^3$

$w_0 = \rho_0 g = 2400 \times 9.81 = 23544 \, N/m^3$

Now $p = \frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]$

$= \frac{15696 \times 10^2}{2}\left[\frac{1-\sin 30°}{1+\sin 30}\right]$

$p = \cancel{784800}$ 261600 N

w.k.t

$W = \frac{W_0 h (a+b)}{2}$

$= \frac{23544 \times 10 (2+8)}{2}$

$W = 1177200 \, N$

$d = AN + x$

But $AN = \frac{a^2+ab+b^2}{3(a+b)} = \frac{2^2 + 2\times8 + 8^2}{3(2+8)} \Rightarrow 2.8$

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$x = \frac{Pb}{3W}$

$= \frac{261600 \times 10}{3 \times 1177200}$

$x = 0.74$ m

$d = AN + x$

$= 2.8 + 0.74$

$d = 3.54$ m

$e = d - b/2$

$= 3.54 - \frac{8}{2}$

$e = -0.46$ $\therefore e = 0.46$. (numerically).

Now

$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]$

$= \frac{1177200}{8}\left[1 + \frac{6 \times (0.46)}{8}\right]$

$\boxed{\sigma_{max} = 197916.75}$ N/m² (comp)

$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$

$= \frac{1177200}{8}\left[1 - \frac{6 \times 0.46}{8}\right]$

$\boxed{\sigma_{min} = 96383.25}$ N/m² (comp)

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3) A Masnory retaining wall of Trapezoidal section 2m wide at its top, 3m wide at its bottom is 8m height. It is retaining a soil on its vertical side at a surcharge of 20°. The soil has a density of 2000 kg/m3. and has an angle of repose 45°. find the total pressure on the wall per metre length. and the point where the resultant cuts the base. Also find maximum and minimum intensities of stress at base. Take the density of Masnory has 2400 kg/m³.

Soln:- Given data:-

$h = 8m$

$a = 2m$

$b = 3m$

$\alpha = 20°$

$\phi = 45°$

$\rho_0 = 2400 \cdot kg/m^3$

$\rho = 2000 kg/m^3$

w.k.t.

$w = \rho \times g = 2000 \times 9.81 = 19620 \ N/m^3$

$w = \rho_0 \times g = 2400 \times 9.81 = 23544 \ N/m^3$.

Now,

$$p = \frac{wh^2}{2} \cos\alpha \left\{ \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha + \cos^2\phi}} \right.$$

$$= \frac{19620 \times 8^2}{2} \cos 20° \times \frac{\cos 20° - \sqrt{\cos^2 20 - \cos^2 45}}{\cos 20° - \sqrt{\cos^2 20 + \cos^2 45}}$$

$P = 89457.95 N$

Now

$P_H = p\cos\alpha = 8945795 \cos 20° = 8406297 N$

$P_V = p\sin\alpha = 8945795 \sin 20° = 30596.42 N$

Now,

$$W = \frac{wh(a+b)}{2}$$

$$= \frac{23544 \times 8 \times (2+3)}{2}$$

$W = 470880\ N$ $\quad \therefore$ Total $W = 470880 + 30596.42$

$W = 501476.42 N.$

Now,

$d = AN + x.$

But $AN = \dfrac{a^2 + ab + b^2}{3(a+b)}$

$$= \frac{2^2 + 3 \times 2 + 3^2}{3(2+3)}$$

$AN = 1.26\ m$

Now,

$$x = \frac{Ph}{3W}$$

$$= \frac{89457.95 \times 8}{3 \times \cancel{470880}\ 501476.42}$$

$x = 0.50m.\ 0.47m$

Now, $d = AN + x.$

$= 1.26 + 0.47$

$d. = 1.73\ m$

Now, $e = d - b/2 \Rightarrow 1.73 - \frac{3}{2}$

$e = 0.23\ m$

Now

$$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]$$

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$$\sigma_{max} = \frac{501476.42}{3}\left[1 + \frac{6 \times 0.23}{3}\right]$$

$$\boxed{\sigma_{max} = 244051.857\ N/m^2} \text{ (comp)}$$

Now

$$\sigma_{min} = \frac{501476.42}{3}\left[1 - \frac{6 \times 0.23}{3}\right]$$

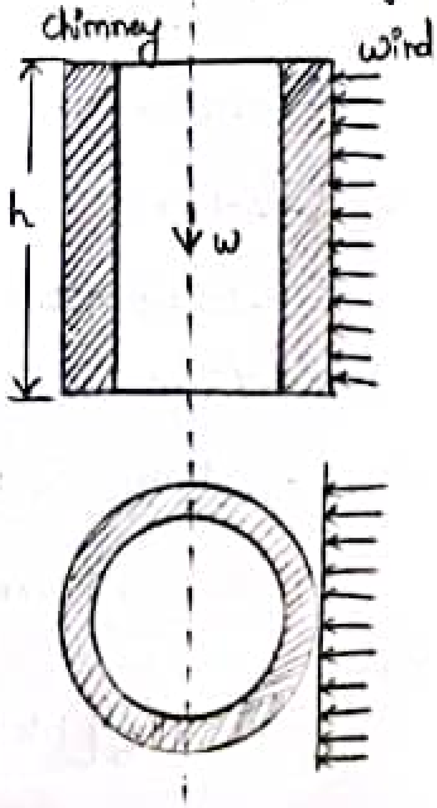
$$\boxed{\sigma_{min} = 90265.75\ N/m^2} \text{ (comp)}$$

* Chimneys :-

Chimneys are tall structures subjected to horizontal wind pressure. the base of the chimneys are subjected to bending moment due to horizontal wind force. This bending moment at the base produces bending stresses. The base of the chimney is also subjected to direct stresses due to self weight of the chimney. Hence at the base of the chimney, the bending stresses and direct stress are acting. The direct stress σ_0 is given by.

$$\sigma_0 = \frac{\text{weight of the chimney}}{\text{Area of section of the base}}$$

$$\sigma_0 = \frac{W}{A}$$



The bending stresses (σ_b) is obtained from

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M}{I} \times y \Rightarrow \frac{M}{(I/y)} \Rightarrow \frac{M}{Z}$$

where M = Bending moment due to horizontal wind force and

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Z = modulus of section

The wind force (F) acting in the horizontal direction on the surface of chimney is given by

$$F = K \times p \times A.$$

where K = coefficient of wind resistance, which depends upon the shape of the area exposed to wind.

= 1 for rectangular and square chimneys

= $\frac{2}{3}$ for circular chimney.

p = intensity of wind pressure

A = projected area of the surface exposed to wind

= D×h for circular chimney.

= b×h for rectangular and square chimney

b = width of chimney exposed to wind.

h = height of chimney.

The wind force F will be acting at $h/2$.

The moment of F at the base of the chimney will be $F \times \frac{h}{2}$

Hence bending moment (M) at the base of chimney is given by,

$$\boxed{M = F \times \frac{h}{2}}$$

Q.) Determine the maximum and minimum stresses at the base of an hollow circular chimney of height 20m with external diameter 4m and internal diameter 2m. The chimney is subjected to a horizontal wind pressure of intensity $1 kN/m^2$. The specific weight of the material of the chimney is $22 kN/m^3$.

soln:-

Given:-

$h = 20m$

$D_o = 4m$

$D_i = 2m$

$p = 1\ kN/m^2$

$w = 22\ kN/m^3$

w.k.t $\sigma_{max} = \sigma_d + \sigma_b$

But

$$\sigma_d = \frac{W}{A}$$

Now, $W = $ wt. density of $\times$ vol

$$W = w \times A \times h$$

$$= w \times \left[\frac{\pi}{4}\left(D_o^2 - D_i^2\right)\right] \times h$$

$$= 22 \times \left[\frac{\pi}{4}\left(4^2 - 2^2\right)\right] \times 20$$

$W = 4146.9\ kN.$

Now

$$A = \frac{\pi}{4}\left[D_o^2 - D_i^2\right]$$

$$A = \frac{\pi}{4}\left[4^2 - 2^2\right]$$

$A = 9.424\ m^2$

Now

$$\sigma_d = \frac{W}{A}$$

$$= \frac{4146.9}{9.424}$$

$\sigma_d = 440\ kN/m^2.$

Now, $\sigma_d = 440\ kN/m^2.$

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Now

$\sigma_b = \frac{M}{Z}$

But, $M = F \times \frac{h}{2}$

And,

$F = K \times p \times A$.

for circular chimney $K = \frac{2}{3}$

& $A = D \times h$.

$= D_o h$

$= 4 \times 20$

$A = 80\ m^2$

Now, $F = \frac{2}{3} \times 1 \times 80$

$F = 53.33$ kN.

Now, $M = F \times \frac{h}{2}$

$= 53.33 \times \frac{20}{2}$

$M = 533.3$ kN-m

Now, $Z = \frac{I}{y}$

$= \frac{\frac{\pi}{64}[D_o^4 - D_i^4]}{D_o/2}$

$= \frac{\frac{\pi}{64}[4^4 - 2^4]}{4/2}$

$Z = 5.89\ m$

Now,

$\sigma_b = \frac{M}{Z} = \frac{533.3}{5.89} = 90.54\ kN/m^2$.

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NOW

$\sigma_{max} = \sigma_d + \sigma_b = 440 + 90.54$

$\sigma_{max} = 530.54 \ kN/m^2$. (comp)

$\sigma_{min} = \sigma_d - \sigma_b = 440 - 90.54$

$\sigma_{min} = 349.46 \ kN/m^2$ (comp)

MODULE-III

THIN CYLINDERS & THICK CYLINDERS

UNIT - IV

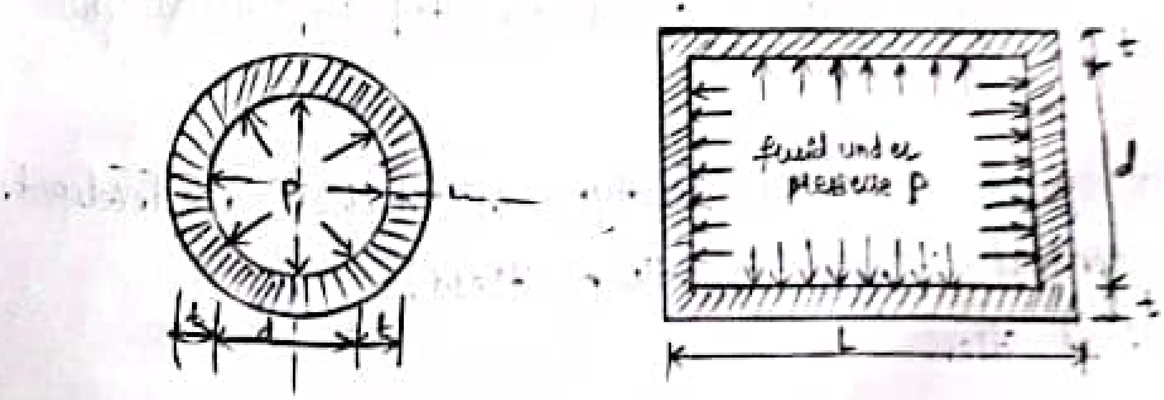
Thin cylinders and thick cylinders.

* Introduction :-

The vessels such as boilers, compressed air receiver, etc. are of cylindrical and spherical form. These vessels are generally used for storing fluids (liquids or gases) under pressure. The walls of such vessels are thin as compared to their diameter. If the thickness of the walls of cylindrical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the cylindrical vessel is known as thin cylinder. In case of thin cylinder, the stress distribution is assumed to be uniform over the thickness of the wall.

* Thin cylindrical vessel subjected to internal pressure :-

fig. shows a thin cylinderical vessel in which fluid under pressure is stored.



let d = internal diameter of thin cylinder.
t = thickness of the wall of the cylinder.
p = internal pressure of the fluid.
L = length of the cylinder.

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on the account of internal pressure p, the cylindrical vessel may fail by splitting up in any one of the two ways as shown in figure.(a) & fig (b)

The forces due to pressure of the fluid acting vertically upwards and downwards on the cylinder, tends to burst the cylinder as shown in fig(a).

The forces due to pressure of the fluid, acting at the ends of the cylinder, tends to burst the cylinder as shown in fig. (b).

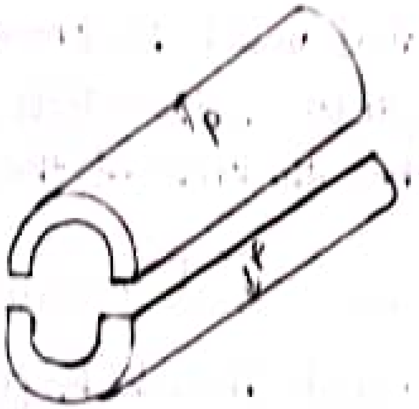

fig (a)

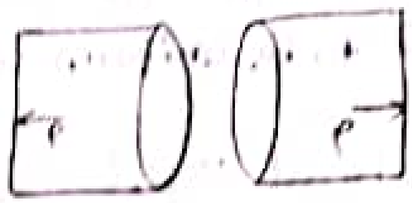

fig (b).

* Stresses in a thin cylindrical vessel subjected to internal fluid pressure:-

In thin cylindrical vessel two types of stresses are developed.

1) Circumferential stress. (or) Hoop stress.
2) Longitudinal stress.

1 * Circumferential stress:-

The name of the stress is given according to the direction in which it is acting. The stress acting along the

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circumference of the cylinder is called circumferential stress.

) longitudinal stress:-

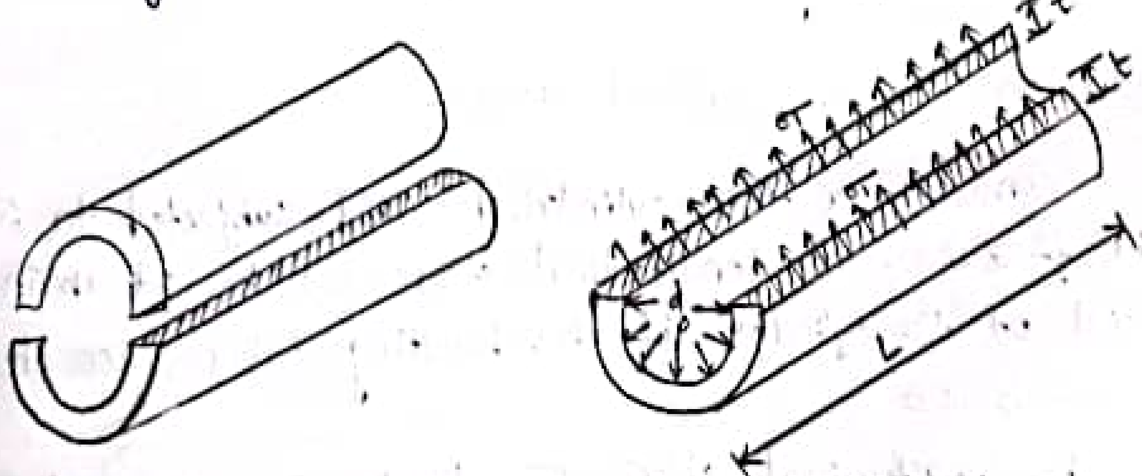
The stress acting along the length of the cylinder (longitudinal direction) is known as longitudinal stress.

* Expression for circumferential stress (or) Hoop stress:-

consider a thin cylindrical vessel subjected to an internal fluid pressure., the circumferential stress will be set up in the material of the cylinder, if bursting of the cylinder takes place as shown in the figure.

The expression for hoop stress or circumferential stress is σ_1 is obtained as given below.

The bursting will takes place.



The bursting will takes place if the force due to fluid pressure is more than resisting force due to circumferential stress set up in the material. In the limiting case, two forces should be equal.

Force due to pressure $F = p \times$ Area on which p is acting.

$= p \times A$

$$f = p \times d \times L$$

$$f = pdL \quad \text{—} \ (1)$$

force on due to ~~conferenti~~ ~~cur~~ circumferential stress,

$f = \sigma_1 \times$ Area on which σ_1 is acting.

$$f = \sigma_1 \times A$$

$$f = \sigma_1 \times (L \times t + L \times t)$$

$$f = 2\sigma_1 Lt \quad \text{—} \ (2)$$

Equating (1) & (2)

$$pdL = 2\sigma_1 Lt$$

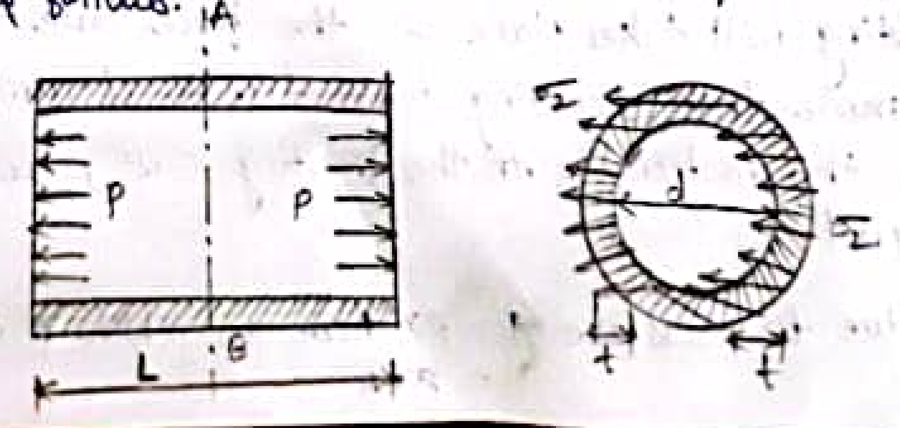
$$\sigma_1 = \frac{pdL}{2Lt}$$

$$\boxed{\sigma_1 = \frac{pd}{2t}}$$

* Expression for longitudinal stress :-

Consider a thin cylindrical vessel subjected to internal fluid pressure. the longitudinal stress will be set up in the material of the cylinder. if the bursting takes place along the section AB.

The longitudinal stress σ_2 developed in the material is as follows.



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The bursting will takes place if the force due to fluid pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal stress σ_2 developed in the material as shown in figure. In limiting case both the forces should be equal.

force due to pressure $f = p \times A$.

$$f = p \times \frac{\pi}{4} d^2$$

$$f = \frac{p \times \pi d^2}{4} \longrightarrow ①$$

force due to longitudinal stress, $F = \sigma_2 \times A$.

$$f = \sigma_2 \times \pi d \times t$$

$$f = \sigma_2 \pi d t$$

$$f = \sigma_2 \pi d t \longrightarrow ②$$

Equating ① and ②

$$\frac{p \cancel{\pi} d^{\cancel{2}}}{4} = \sigma_2 \cancel{\pi} \cancel{d} t$$

$$\boxed{\sigma_2 = \frac{pd}{4t}}$$

$$\sigma_2 = \frac{1}{2} \times \frac{pd}{2t}$$

$$\sigma_2 = \frac{1}{2\sigma_1}$$

$$\boxed{\sigma_2 = \frac{\sigma_1}{2}}$$

Hence, circumferential stress is two times that of longitudinal stress hence in the material of the cylinder the permissiable stress should be less than circumferential stress or circumferential stress should not be greater than permissible stress in the material.

* **Maximum shear stress:-**

At any point in the material of the cylindrical shell there are two cylind principle stress mam namely circumference stress $\sigma_1 = \frac{pd}{2t}$ acting circumferentially and longitudinal stress $\sigma_2 = \frac{pd}{4t}$ acting parallel to the axis of the shell. These two stress are co tensile and perpendicular to each other.

$\therefore$ Maximum shear stress $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

$$= \frac{\frac{2pd - pd}{4t}}{2}$$

$$\boxed{\tau_{max} = \frac{pd}{8t}}$$

* **Note:-**

1) If the maximum permissible stress in the material is given. this stress is taken as circumferential stress σ_1.

2) If the thickness of the thin cylinder to be determined than circumferential stress eqn i.e $\sigma_1 = \frac{pd}{2t}$ should be used.

1) A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to a internal fluid pressure of 1.2 N/mm². Determine
i) longitudinal stress developed in the pipe,
ii) circumferential stress developed in the pipe.

Soln:- Given. $d = 1.5m = 1500\,mm$

$t = 1.5\,cm = 15\,mm$

$p = 1.2\ N/mm^2$

i) w.k.t

$$\sigma_2 = \frac{pd}{4t}$$

$$= \frac{1.2 \times 1500}{4 \times 15}$$

$$\sigma_2 = 30\ N/mm^2$$

ii) w.k.t

$$\sigma_1 = \frac{pd}{2t}$$

$$= \frac{1.2 \times 1500}{2 \times 15}$$

$$\sigma_1 = 60\ N/mm^2$$

2) A cylinder of internal diameter 2.5m and thickness 5cm contains a gas. if the tensile stress in the material is not to exceed $80\ N/mm^2$. Determine the internal pressure of gas.

Sol^n $d = 2.5m = 2500\ mm$

$t = 5cm = 50\ mm$

$\sigma_1\ \cancel{\sigma_{max}} = 80\ N/mm^2$. (Maximum permissible stress)

$p = ?$

w.k.t $\sigma_1 = \frac{pd}{2t}$

$$80 = \frac{p \times 2500}{2 \times 50}$$

$$p = 3.2\ N/mm^2$$

3) A cylinder of internal diameter 0.5m contains air at a pressure of $7\ N/mm^2$. If the maximum permissible stress induced in the material is $80\ N/mm^2$, find the thickness of the cylinder

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Soln:- Given

$d = 0.5\,m = 500\,mm$

$p = 7\,N/mm^2$.

$\sigma_1 = 80\,N/mm^2$ (Maximum permissible stress).

$t = ?$

w.k.t

$$\sigma_1 = \frac{pd}{2t}$$

$$80 = \frac{7 \times 500}{2 \times t}$$

$t = 21.875\,mm$.

As maximum permissible stress is indirectly proportional to thickness as thickness increases the stress value decreases and vice versa. As the given stress should not exceed $80\,N/mm^2$, so the thickness should not be ~~inc~~ decreased and it should be increased or keep it unchanged.

∴ thickness $\boxed{t = 22\,mm}$

4) A thin cylinder of internal diameter 1.25 m contains a fluid at an internal pressure of $2\,N/mm^2$. Determine the maximum thickness of the cylinder if.

i) Longitudinal stress is not to exceed. $30\,N/mm^2$.

ii) Circumferential stress is not to exceed $45\,N/mm^2$.

Soln:- Given $d = 1.25\,m = 1250\,mm$

$p = 2\,N/mm^2$

i) longitudinal stress $\sigma_2 = 30\,N/mm^2$

w.k.t $\sigma_2 = \frac{pd}{4t}$.

$20 = \frac{2 \times 1200}{4 \times t}$

$t = 20.83$ mm.

ii) circumferential stress $\sigma_1 = \frac{pd}{2t}$

$45 = \frac{2 \times 1250}{2 \times t}$

$t = 27.77$ mm

∴ The longitudinal and circumferential stresses induced in the material are inversely proportional to the thickness 't' of the cylinder. Hence the stress induced will be less if the value of 't' is more. Hence take the maximum value of 't' from (i) and (ii).

∴ $t = 27.77 \simeq 28$ mm.

5) - A water main 80cm diameter contains a water at a pressure head of 100m. If the weight density of the water is 9810 N/m³, find the thickness of the metal required for the water main, Given the permissible stress of 20 N/mm².

Sol^n: Given

$d = 80$ cm $= 800$mm

$h = 100$m

$w = 9810$ N/m³.

$\sigma_1 = 20$ N/mm²

w.k.t $\sigma_1 = \frac{pd}{2t}$

But $p = wh = 9810 \times 100$

$p = 981000$ N/m²

$p = 981000 \times 10^{-6}$ N/mm²

$p = 0.981\ N/mm^2$

Now,

$$\sigma_1 = \frac{pd}{2t}$$

$$20 = \frac{0.981 \times 800}{2t}$$

$t = 19.62$ mm

$t = 20$ mm

* Efficiency of a joint :-

The cylinderical shells such as boilers are having two types of joints namely longitudinal joint and the circumferential joint. In case of a joint, holes are made in the material of the shell for the rivets. Due to the holes, the area offering resistance decreases. Due to decrease in the area, the stress developed in the material of the shell will be more.

Hence in the case of rivetted shell, the circumferential and longitudinal stresses are greater than the stresses which are derived previously. If the efficiency of a longitudinal joint and circumferential joint are given then the circumferential and longitudinal stress are obtained as follows.

let η_l = efficiency of longitudinal joint. (η_L)

η_c = efficiency of circumferential joint.

Then, the circumferential joint stress $\sigma_1 = \dfrac{pd}{2t\eta_l}$

and, the longitudinal stress $\sigma_2 = \dfrac{pd}{4t\eta_c}$

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Note:-

1. In longitudinal joint, circumferential stress is developed, whereas in circumferential joint, the longitudinal stress is developed.

2. Efficiency of a joint means efficiency of a longitudinal joint.

3. If the efficiency of a joint is given, the thickness of thin shell is determined from. eqn $\sigma_1 = \frac{pd}{2t\eta_l}$

i) A boiler is subjected to an internal steam pressure of $2\ N/mm^2$ the thickness of boiler plate is 2 cm and permissible tensile stress is $120\ N/mm^2$. Find out the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.

Soln- Given.

$p = 2\ N/mm^2$.

$t = 2\,cm = 20\,mm$

permissible stress $= 120\ N/mm^2$.

$\eta_c = 40\% = \frac{40}{100} = 0.4$

$\eta_l = 90\% = \frac{90}{100} = 0.9$.

i) taking permissible stress $= \sigma_1$

$\therefore$ w.k.t $\sigma_1 = \frac{pd}{2t\eta_l}$

$$120 = \frac{2 \times d}{2 \times 20 \times 0.9}$$

$d = 2160$ mm

ii) Taking permissible stress $= \sigma_2$

w.k.t $\sigma_2 = \frac{pd}{4t\eta_c}$

$$120 = \frac{2\times d}{4\times 20\times 0.4}$$

$d = 1920$ mm.

∴ The longitudinal or circumferential stresses induced in the material are directly proportional to diameter d. Hence, the stress induced will be less if the value of d is less.

Hence take the minimum value of d calculated from (i) and (ii).

∴ Maximum diameter of the boiler is equal to minimum value of diameter given by (i) & (ii).

∴ Maximum diameter $d = 1920$ mm.

2) A boiler shell is to be made of 15 mm thick plate having limiting tensile stress of 120 N/mm². If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively. Determine

i) The maximum permissible diameter of the shell for an internal pressure of 2 N/mm².

ii) permissible intensity of internal pressure when the shell diameter is 1.5 m.

Soln:- Given $t = 15$ mm

permissible stress = 120 N/mm²

$$\eta_l = 70\% = \frac{70}{100} = 0.7$$

$$\eta_c = 30\% = \frac{30}{100} = 0.3$$

i) Given $p = 2$ N/mm².

a) Taking permissible stress = σ_T

w.k.t $\sigma_1 = \frac{pd}{2t\eta_l}$

$120 = \frac{2 \times d}{2 \times 15 \times 0.7}$

$d = 1260$ mm

b) Taking permissible stress $= \sigma_2$

w.k.t

$\sigma_2 = \frac{pd}{4t\eta_c}$

$120 = \frac{2 \times d}{4 \times 15 \times 0.3}$

$d = 1080$ mm

Thus the maximum diameter of the shell which is satisfying both the conditions is $d = 1080$ mm.

ii) Given. $d = 1.5$ m $= 1500$ mm

a) w.k.t

$\sigma_1 = \frac{pd}{2t\eta_l}$

$120 = \frac{p \times 1500}{2 \times 15 \times 0.7}$

$p = 1.68$ N/mm².

b) w.k.t $\sigma_2 = \frac{pd}{4t\eta_c}$

$120 = \frac{p \times 1500}{4 \times 15 \times 0.3}$

$p = 1.44$ N/mm²

Hence in order to satisfy both the conditions, the maximum permissible internal pressure is equal to the minimum value of the pressure given by (a) and (b).

∴ Maximum permissible internal pressure = 1.44 N/mm^2.

3) A cylinder of thickness 1.5 cm has to withstand maximum internal pressure of 1.5 N/mm^2. If the ultimate tensile stress in the material of the cylinder is 300 N/mm^2, factor of safety is 3 and joint efficiency is 80%, Determine the diameter of the cylinder.

Sol:- $t = 1.5m = 15mm.$ $\quad \therefore \eta\% = \frac{80}{100} = 0.8$

$$\text{working stress} = \frac{\text{ultimate stress}}{\text{factor of safety}}$$

$$= \frac{300}{3}$$

$$\text{working stress} = 100 = \sigma$$

$$\sigma_2 = \frac{pd}{4t\eta_c}$$

$$100 = \frac{1.5 \times d}{4 \times 15 \times 0.8}$$

$$d = \frac{4 \times 100 \times 15 \times 0.8}{1.5}$$

$$d = 3200\,mm$$

$$\sigma_1 = \frac{pd}{2t\eta_L}$$

$$100 = \frac{1.5 \times d}{2 \times 15 \times 0.8}$$

$$d = \frac{2 \times 100 \times 15 \times 0.8}{1.5}$$

$$\boxed{d = 1600\,mm}$$

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* Effect of internal pressure on the dimensions of a thin cylindrical shell:-

when a fluid having internal pressure p is stored in a thin cylindrical shell. due to internal pressure of the fluid, the stresses set up at any point of the material of the shell are

1) Hoop or circumferential stress σ_1 acting on longitudinal section.

2) Longitudinal stress σ_2 acting on circumferential section.

These stresses are principle stress, as they are acting on principle planes. The stress in the 3rd principle plane is zero. as the thickness 't' of the cylinder is very small and it is radial stress which can be neglected.

Now $\sigma_1 = \frac{Pd}{2t}$

$\sigma_2 = \frac{pd}{4t}$

circumferential strain

$$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{Pd}{2tE} - \mu\frac{Pd}{4tE}$$

$$e_1 = \frac{Pd}{2tE}\left[1-\frac{\mu}{2}\right] \longrightarrow ①$$

But e_1 = circumferential strain

$$e_1 = \frac{\text{change in circumference}}{\text{original circumference}}$$

$$e_1 = \frac{\text{final circumference} - \text{initial circumference}}{\text{original circumference}}.$$

$$e_1 = \frac{\pi(d+\delta d) - \pi d}{\pi d}$$

$$e_1 = \frac{\cancel{\pi d} + \pi\delta d - \cancel{\pi d}}{\pi d}$$

$$e_1 = \frac{\cancel{\pi}\delta d}{\cancel{\pi} d}$$

$$e_1 = \frac{\delta d}{d} \longrightarrow ②$$

Now ① = ②

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\boxed{\delta d = \frac{pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]}$$

longitudinal strain:-

$$e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$e_2 = \frac{pd}{4tE} - \mu\frac{pd}{2tE}$$

$$e_2 = \frac{pd}{2tE}\left[\frac{1}{2} - \mu\right] \longrightarrow ③$$

But,

e_2 = longitudinal strain

$$e_2 = \frac{\text{change in length}}{\text{original length}}$$

$$e_2 = \frac{(L+\delta L) - L}{L}$$

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$$e_2 = \frac{\cancel{L} + \delta L - \cancel{L}}{L}$$

$$e_2 = \frac{\delta L}{L} \longrightarrow (4)$$

(3) = (4)

$$\frac{\delta L}{L} = \frac{pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$\boxed{\delta L = \frac{pdL}{2tE}\left[\frac{1}{2} - \mu\right]}$$

~~vol~~ <u>Volumetric</u> <u>strain</u>:-

It is defined as change in volume to the original volume.

so. i.e $e_v = \frac{\delta V}{V}$

But δV = change in volume

δv = final vol − initial vol.

But Initial / original volume, $V = \frac{\pi}{4} d^2 L$

Now final volume $= \frac{\pi}{4}(d+\delta d)^2 [L + \delta L]$

$$= \frac{\pi}{4}\left[d^2 + (\delta d)^2 + 2d\delta d\right]\left[L + \delta L\right]$$

$$= \frac{\pi}{4}\left[d^2 L + (\delta d)^2 L + 2d\delta d L + d^2 \delta L + (\delta d)^2 \delta L + 2d\delta d \delta L\right]$$

final volume $= \frac{\pi}{4}\left[d^2 L + 2dL\delta d + \delta L d^2\right]$

Now,

δv = final vol − initial vol.

$$= \frac{\pi}{4}\left[d^2 L + 2dL\delta d + \delta L d^2\right] - \frac{\pi}{4} d^2 L$$

$$= \cancel{\frac{\pi}{4} d^2 L} + \frac{\pi}{4} 2dL\delta d + \frac{\pi}{4} \delta L d^2 - \cancel{\frac{\pi}{4} d^2 L}$$

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$$\delta V = \frac{\pi}{4}\left[2dL\delta d + \delta L d^2\right]$$

Now

$$V = \frac{\pi}{4}d^2L$$

$$\frac{\delta V}{V} = \frac{\cancel{\frac{\pi}{4}}\left[2dL\delta d + \delta L d^2\right]}{\cancel{\frac{\pi}{4}}d^2L}$$

$$\frac{\delta V}{V} = \frac{2dL\delta d}{d^2L} + \frac{\delta L d^2}{d^2L}$$

$$\frac{\delta V}{V} = \frac{2\delta d}{d} + \frac{\delta L}{L}$$

$$\frac{\delta V}{V} = 2e_1 + e_2$$

$$= 2\left[\frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)\right] + \frac{pd}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{pd}{2tE}\left[2-\frac{2\mu}{2}+\frac{1}{2}-\mu\right]$$

$$= \frac{pd}{2tE}\left[2+\frac{1}{2}-2\mu\right]$$

$$\frac{\delta V}{V} = \frac{pd}{2tE}\left[\frac{5}{2}-2\mu\right]$$

$$\therefore \boxed{\delta V = \frac{pdV}{2tE}\left[\frac{5}{2}-2\mu\right]}$$

1) Calculate i) change in diameter
ii) change in length.
iii) change in volume of a thin cylindrical shell, 100cm diameter, 1cm thick and 5m long when subjected to a internal pressure of 3 N/mm². Take $E = 2\times10^5$ N/mm² and poisson

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ratio $\mu = 0.3$.

Sol:

$d = 100\,cm = 1000\,mm$

$t = 1\,cm = 100\,mm$

$l = 5\,m = 5000\,mm$

$p = 3\,N/mm^2$.

$E = 2\times10^5\,N/mm^2$

$\mu = 0.3$

i) w.k.t

$$\delta d = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$\delta d = \frac{3\times1000^2}{2\times10\times2\times10^5}\left[1-\frac{0.3}{2}\right]$$

$\delta d = 0.6375\,mm$

ii) w.k.t

$$\delta L = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{3\times1000\times5000}{2\times10\times2\times10^5}\left[\frac{1}{2}-0.3\right]$$

$\delta L = 0.75\,mm$

iii) w.k.t

$$\delta V = \frac{pdV}{2tE}\left[\frac{5}{2}-2\mu\right]$$

$\because$ But $V = \frac{\pi}{4}d^2L \Rightarrow \frac{\pi}{4}[1000]^2\times5000$

$V = 3926990817\ mm^3$.

$$= \frac{3\times1000\times3926990817}{2\times10\times2\times10^5}\left[\frac{5}{2}-2\times0.3\right]$$

$\delta V = 5595962.44\,mm^3$ //

Q.2) A cylindrical shell 90cm long, 20 cm internal diameter having thickness of metal has 8mm is filled with a fluid at atmospheric pressure. If an additional 20 cm^3 of fluid is pumped into the cylinder, find the pressure exerted by the fluid on the cylinder and Hoop stress induced. Take $E = 2\times10^5 \ N/mm^2$ and $\mu = 0.3$

Soln:- Given.

$l = 90cm = 900mm$

$d = 20cm = 200mm$

$t = 8mm$

$\delta V = 20 cm = 20\times10^3 \ mm^3$

$E = 2\times10^5 \ N/mm^3$

$\mu = 0.3$

w.k.t

$$\delta V = \frac{pdV}{2tE}\left[\frac{5}{2} - 2\mu\right]$$

$$= \frac{p\times200\times28274333.88\left[\frac{5}{2} - 2\times0.3\right]}{2\times8\times2\times10^5}$$

but $V = \frac{\pi}{4}d^2l = \frac{\pi}{4}(200)^2\times900$

$V = 28274333.88 \ mm^3.$

$$20\times10^3 = \ldots$$

$$\frac{20\times10^3\times2\times8\times2\times10^5}{200\times28274333.88\left[\frac{5}{2} - 2\times0.3\right]} = p$$

$$p = 5.95 \ N/mm^2$$

Now

Hoop stress, $\sigma_1 = \frac{pd}{2t}$

$$= \frac{5.95\times200}{2\times8}$$

$$\sigma_1 = 74.37 \ N/mm^2$$

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Q.3) A cylindrical shell 3m long which is ~~cooled~~ closed at the ends of an internal diameter of 1m and ~~above~~ a wall thickness of 15 mm. calculate the circumferential and longitudinal stresses induced and also changes in the dimensions of the ~~ch~~ shell. if it is subjected to an internal pressure of 1.5 N/mm². Take $E = 2\times10^5$ N/mm² and $\mu = 0.3$.

Soln:- Given.

$L = 3\,m = 3000\,mm$

$d = 1\,m = 1000\,mm$

$t = 15\,mm.$

$p = 1.5\,N/mm^2.$

$E = 2\times10^5\,N/mm^2$

$\mu = 0.3$

Now.

i) circumferential stress σ_1

$$\sigma_1 = \frac{pd}{2t}$$

$$= \frac{1.5\times1000}{2\times15}$$

$$\sigma_1 = 50\,N/mm^2$$

longitudinal stress σ_2.

$$\sigma_2 = \frac{pd}{4t} = \frac{1.5\times1000}{4\times15}$$

$$\sigma_2 = 25\,N/mm^2.$$

Now.

$$\delta L = \frac{pdL}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{1.5\times1000\times3000}{2\times15\times2\times10^5}\left[\frac{1}{2}-0.3\right]$$

$\delta L = \cancel{0.0125\,mm}$ 0.15 mm.

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$= \frac{1.5 \times 1000^2}{2 \times 15 \times 2 \times 10^5}\left[1 - \frac{0.3}{2}\right]$$

$\delta d = 0.2125$ mm

$$\delta v = \frac{pdv}{2tE}\left[\frac{5}{2} - 2\mu\right]$$

$\because V = \frac{\pi}{4}d^2 l = \frac{\pi}{4} \times (1000)^2 \times 3000$

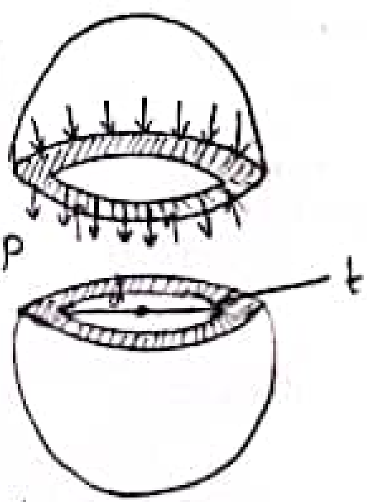
$V = 2356194490$

$$= \frac{1.5 \times 1000 \times 2356194490}{2 \times 15 \times 2 \times 10^5}\left[\frac{5}{2} - 2 \times 0.3\right]$$

$\delta v = 1119192.383$ mm^3

* Thin spherical shells:-

figure shows a thin spherical shell of internal diameter 'd' and thickness 't' subjected to an internal fluid pressure 'p'. The fluid inside the shell has a tendency to split the shell into two hemisphere along x-x axis.



The force P which has tendency to split the shell equal to $p \times \frac{\pi}{4}d^2$.

The ~~8~~ resisting area of this force $= \pi dt$.

Therefore, Hoop or circumferential stress σ_1 induced in the material of the shell is given by $\sigma_1 = \frac{\text{force}}{\text{Area}} = \frac{P}{\text{Area resisting the force}}$

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$\sigma_1 = \dfrac{\text{force } P}{\text{Area resisting the force.}}$

$= \dfrac{p \times \frac{\pi}{4} d^2}{\pi d t}$

$= \dfrac{p \times \frac{\pi}{4} d^2}{\pi d t}$

$= \dfrac{p \pi d^2}{4} \times \dfrac{1}{\pi d t}$

$= \dfrac{p \pi d^2}{4 \pi d t}$

$\sigma_1 = \dfrac{pd}{4t}$

∴ The stress σ_1 is tensile in nature.

The fluid inside the shell is dt also having tendency to split the shell into two hemisphere along y-y axis. Then it can be shown that the tensile hoop stress will also be equal to $\frac{pd}{4t}$. let this stress is σ_2

∴ let $\sigma_2 = \dfrac{pd}{4t}$

The stress σ_2 will be at right angle to σ_1

* <u>Change in dimensions of thin spherical shells. due to an internal pressure</u>:-

σ_1 and σ_2 are acting at right angles to each other.

∴ Strain in any direction is given as $e_1 = e_2 = \dfrac{\sigma_1}{E} - \mu\dfrac{\sigma_2}{E}$

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$$e_1 = e_2 = \frac{pd}{4tE} - \frac{\mu pd}{4tE}$$

$$e_1 = e_2 = \frac{pd}{4tE}[1-\mu]$$

But w.k.t strain in any direction is also equal to $e_1 = e_2 = \frac{\delta d}{d}$.

$$e_1 = e_2 = \frac{\delta d}{d}$$

$$\frac{\delta d}{d} = \frac{pd}{4tE}[1-\mu]$$

$$\boxed{\delta d = \frac{pd^2}{4tE}[1-\mu]}$$

Volumetric strain :-

It is defined as the ratio of change in volume to the original volume.

$$e_V = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$$

original volume $V = \frac{4}{3}\pi r^3$.

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$V = \frac{4\pi d^3}{24}$$

$$V = \frac{\pi d^3}{6}$$

Now $e_V = \frac{\delta V}{V}$

$$e_V = \frac{dV}{V}$$

But $V = \frac{\pi}{6} d^3$

Now,

Diff above eqn we get

$$dv = \frac{\pi}{6} 3d^2 \times \delta d$$

Now, $e_v = \frac{dv}{v} = \frac{\frac{\pi}{6} 3d^2 \delta d}{\frac{\pi}{6} d^3}$

$$\frac{dv}{v} = \frac{3\delta d}{d}$$

$$\frac{\delta V}{V} = 3\frac{\delta d}{d}$$

$$\frac{\delta V}{V} = 3\frac{pd}{4tE}[1-\mu]$$

$$\boxed{\delta V = \frac{3pdV}{4tE}[1-\mu]}$$

Q) A vessel in a shape of a spherical shell of 1.2 m internal diameter and 12 mm shell thickness is subjected a pressure of 1.6 N/mm². Determine the stress induced in the material of the vessel.

Soln:- Given

$d = 1.2m = 1200 mm$

$t = 12mm$

$p = 1.6 N/mm^2$

w.k.t $\sigma = \frac{pd}{4t}$

$$\sigma = \frac{1.6 \times 1200}{4 \times 12}$$

$$\sigma = 40 N/mm^2$$

2) A spherical vessel 1.5m diameter is subjected to an internal pressure of 2N/mm². find the thickness of the plate required if the maximum stress is not to exceed 150 N/mm². and joint efficiency is 75%

Soln:- Given:-

$d = 1.5m = 1500mm$

$p = 2N/mm^2.$

$\sigma = 150 \ N/mm^2.$

$\eta = 75\% = 0.75$

w.k.t $\sigma = \frac{pd}{4t\eta}$

$150 = \frac{2 \times 1500}{4 \times t \times 0.75}$

$\boxed{t = 6.66 \ mm}$

3) A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of 1.4 N/mm². Determine the increase in diameter and increase in volume. Take $E = 2\times10^5 N/mm^2$ & $\mu = 1/3$

Soln:- Given: $d = 0.9m = 900mm.$

$t = 10 \ mm$

$p = 1.4 N/mm^2$

$E = 2\times10^5 N/mm^2.$ & $\mu = 1/3 = 0.3$

w.k.t

$\delta d = \frac{pd^2}{4tE}[1-\mu]$

$= \frac{1.4 \times 900^2}{4\times10\times2\times10^5}[1-0.3]$

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$$\boxed{\delta d = 0.0945 \text{ mm}}$$

w.k.t

$$\delta V = \frac{3pdV}{4tE}[1-\mu]$$

$$= \frac{3 \times 1.4 \times \pi/6 \times (900)^3}{4 \times 10 \times 2 \times 10^5}[1-0.3]$$

$\delta V = \cancel{38170}$, 120236.60 mm^3.

$$\boxed{\delta V = 120236.60 \text{ mm}^3}$$

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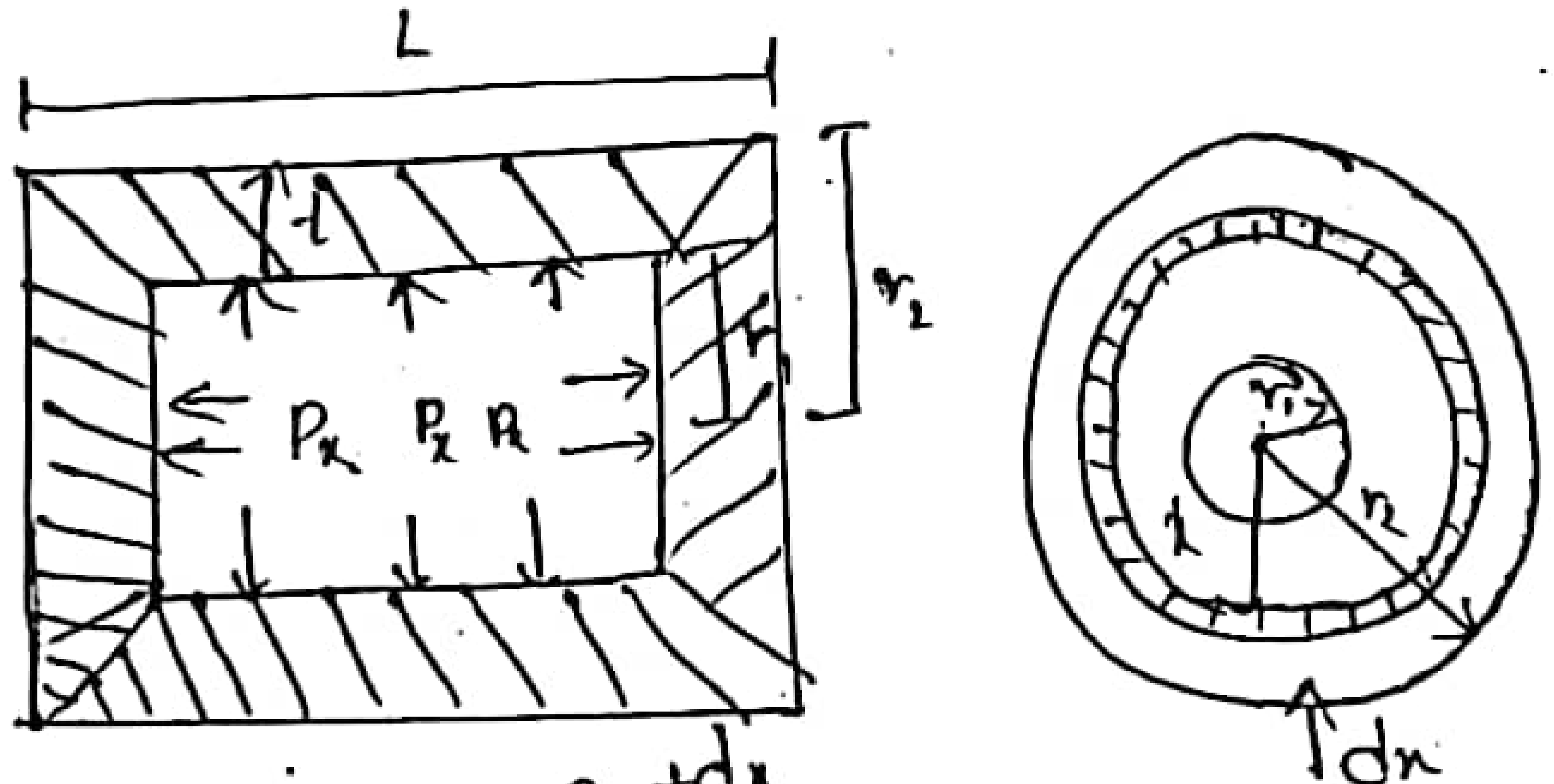
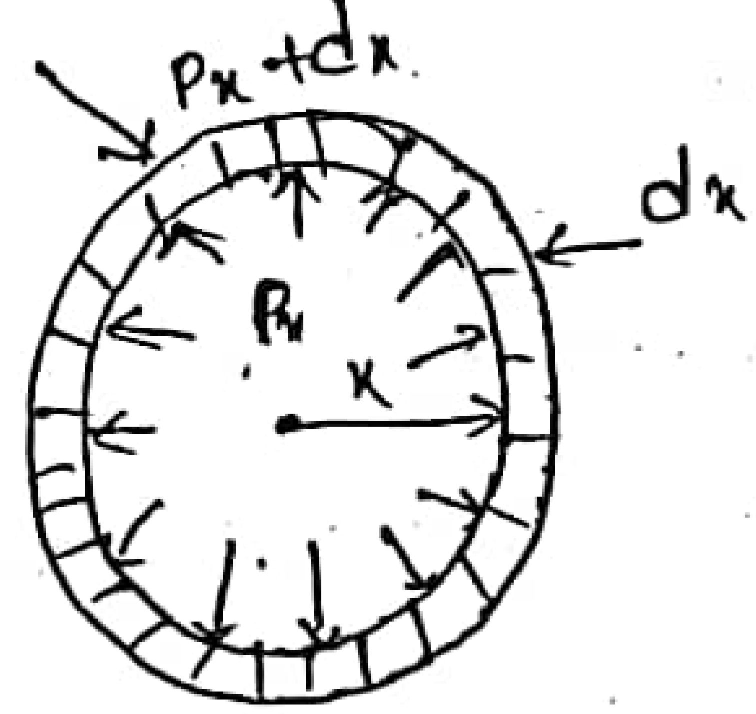
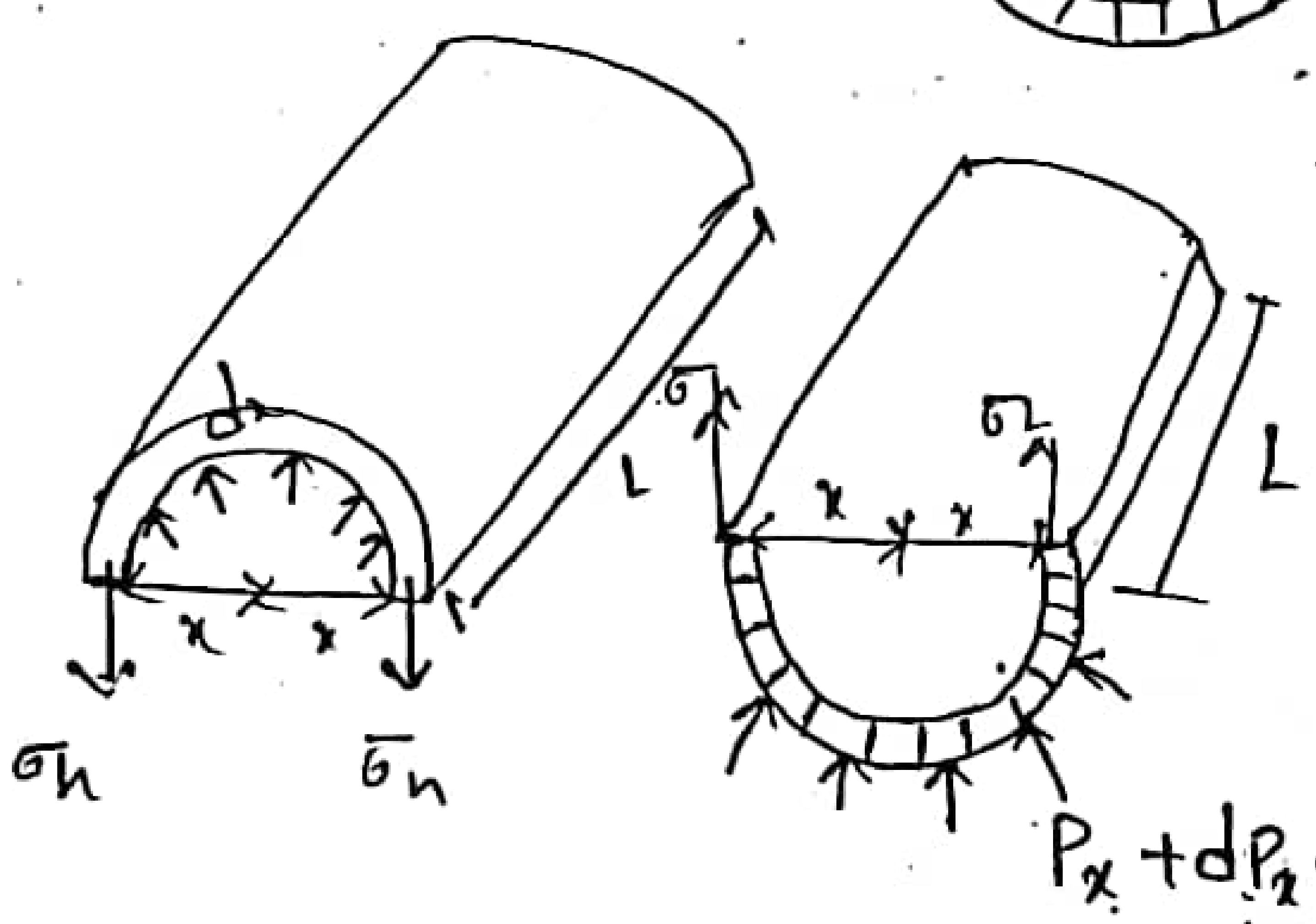
* Thick Cylinder :-





$\frac{t}{D} \leq 20 \Rightarrow$ Thin

$\frac{t}{D} > 20 =$ Thick Cylinder.



Resultant force $= (P_x)(2xL) - (P_x + dP_x)\, 2(x+dx)L$

$= P_x(2xL) - 2[P_x xL + P_x d_x L + x \cdot dP_x \cdot L + dP_x \cdot d_x L]$

$= P_x(2xL) - 2L[P_x \cdot x + P_x \cdot dx + x dP_x + dP_x dx)$

$= P_x\cancel{(2xL)} - P_x\cancel{(2xL)} - 2P_x d_x L - 2x dP_x L$

$= -2P_x d_x L - 2x \cdot dP_x L$

Resultant force $= -2L[P_x dx + x \cdot dP_x]$ — (1)

The Resisting force due to hoop stress

$= \sigma_x (2)(dx) \cdot L$ — (2)

① − ②

$$\sigma_x (\cancel{2})(dx\cancel{l}) = -\cancel{2}\cancel{l}(P_x\,dx + x\,dP_a)$$

$$\sigma_x\, d_x = -P_x\,dx - x\,dP_x$$

$$\sigma_x = -P_x \frac{dx}{dx} - x.\frac{dP_x}{dx}$$

$$\left[\sigma_x = -P_x . x\frac{dP_x}{dx}\right] \text{—③}$$

* consider a thick cylinder external radius r_2 and internal radius r_1 and the length of the cylinder l.

consider an elementary ring of a cylinder radius 'x' thickness 'dx' the pressure acting inside the elementary ring 'P_x' the pressure on the outer layer of the elementary ring is $P_x + dP_x$. In thick cylinders there are three types of stresses will be develop

① Radial compressive stress (P_x) ② hoops stress (σ_x).
③ longitudinal stress (σ_l) and (σ_2)

In thick cylinders the longitudinal strain is constant

∴ longitudinal stress also constant.

$$-P_x \quad \sigma_x, \quad \sigma_2$$
$$\qquad\qquad\qquad \shortparallel$$
$$\qquad\qquad\qquad f_2$$

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu(-P_x)}{E}$$

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu P_x}{E}$$

$$\sigma_1 \quad \sigma_2 \quad \sigma_3$$
$$\shortparallel \quad \shortparallel \quad \shortparallel$$
$$e_1 \quad e_2 \quad e_3$$

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E}$$

$$e_2 = \frac{\sigma_2}{E} = \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_1}{E}$$

$$e_3 = \frac{\sigma_3}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

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As σ_2, μ, and c are constants

$$-\sigma_x + P_x = \text{constant} \quad [\sigma_x \cdot P_x = \text{constant}]$$

Let us assume constant $= 2a$

$$\boxed{\therefore \sigma_x - P_x = 2a} \text{ (ii)} \Rightarrow \sigma_x = P_x + 2a$$

$$\sigma_x = -P_x - \frac{x \cdot dP_x}{dx}$$

$$P_x + 2a = -P_x - \frac{x\, dP_x}{dx}$$

$$\frac{x\, dP_x}{dx} = -P_x - P_x - 2a$$

$$x\frac{dP_x}{dx} = -2(P_x + a) \Rightarrow \cancel{\frac{x}{dx} = -\frac{2(P_x+a)}{dP_x}}$$

$$\left[\frac{dx}{x} = \frac{-dP_x}{2(P_x+a)}\right] \text{ — (5)} \quad \frac{dx}{x} = \frac{-dP_x}{2(P_x+a)}$$

~~Intergate on both side~~

$$\cancel{\int \frac{1}{x}dx = \int \frac{1}{2} \times \frac{-dP_x}{(P_x+a)}}$$

$$\frac{dP_x}{(P_x+a)} = \frac{-2dx}{x}$$

$$\log(P_x + a) = -2\log x + \log b$$

$$\log(P_x + a) = -\log x^2 + \log b$$

$$\log(P_x + a) = \log b/x^2$$

$$P_x + a = b/x^2$$

$$\left[P_x = \frac{b}{x^2} - a\right] \text{ — (6)}$$

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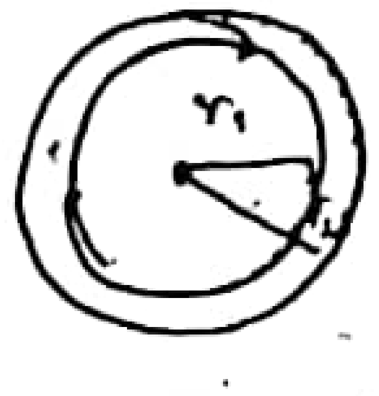
$\sigma_x - P_x = 2a$; $\sigma_x - \left[\frac{b}{x^2} - a\right] = 2a$

$\sigma_x = 2a + \frac{b}{x^2} - a$; $\left[\sigma_x = \frac{b}{x^2} + a\right]$ — (6)

equation (5) and (6) gives the radial stress & hoole's stress at any radius 'x'. & these eq are to known as Lami's equation

An A and B are constant which can be found by using boundaries conditions.

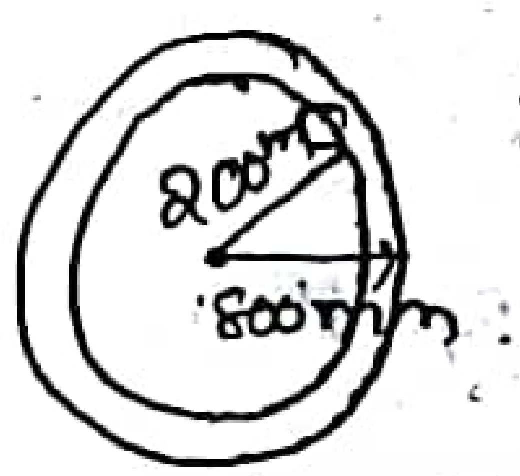
Boundaries Conditions :—



$\{x = r_1 \Rightarrow P_x = P_0$; $x = r_2 \Rightarrow P_x = 0\}$

Determine maximum & min hoops stress across the section of the pipe of 400mm internal dia and 100mm thick when the pipe contains a fluid at a pressure of 800 N/mm² also sketch the radial pressure distribution and hoop stress distrabution

Sol:— Given that,



$d_1 = 400\,mm$, $t = 100\,mm$, $P_0 = 8\,N/mm^2$

$d_2 = 600\,mm$, $r_1 = 200\,mm$, $r_2 = 300\,mm$

P_x = Radial stress , $P_x = \frac{b}{x^2} - a$ — (1)

σ_x = hoopstress , $\sigma_x = \frac{b}{x^2} + a$ — (2)

Boundary conditions

at $x = r_1$; $P_x = P_0$

at $x = 200\,mm$; $P_x = 8\,N/mm^2$

$$8 = \frac{b}{100^2} - a \quad \text{— (3)}$$

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at $x = r_2$; $P_x = 0$

at $x = 300mm$; $P_x = 0$ sub ①

$$0 = \frac{b}{300^2} - a \quad ; \quad a = \frac{b}{300^2} \text{ sub in ③}$$

$$8 = \frac{b}{200^2} - \frac{b}{300^2}$$

$$8 = b\left[\frac{1}{200^2} - \frac{1}{300^2}\right] \Rightarrow 8 = b\left[1.38 \times 10^{-5}\right]$$

$$b = \frac{8}{1.38 \times 10^{-5}} \qquad \boxed{b = 576000}$$

$$\therefore a = \frac{b}{300^2} = \frac{576368}{300^2} = 6.4$$

$$\cancel{\left(P_x + a\right) = \frac{b}{x^2} \Rightarrow P_x = \frac{b}{x^2} - a}$$

$$\cancel{\sigma_x - P_x = 2a}$$

$$\cancel{\sigma_x - \left[\frac{b}{x^2} - a\right] = 2a}$$

$$\cancel{\sigma_x = 2a + \frac{b}{x^2} - a}$$

Radical stress (P_x)

P_x at $r_1 = P_{200} = 8N/mm^2$

P_x at $r_2 = P_{300} = 0$

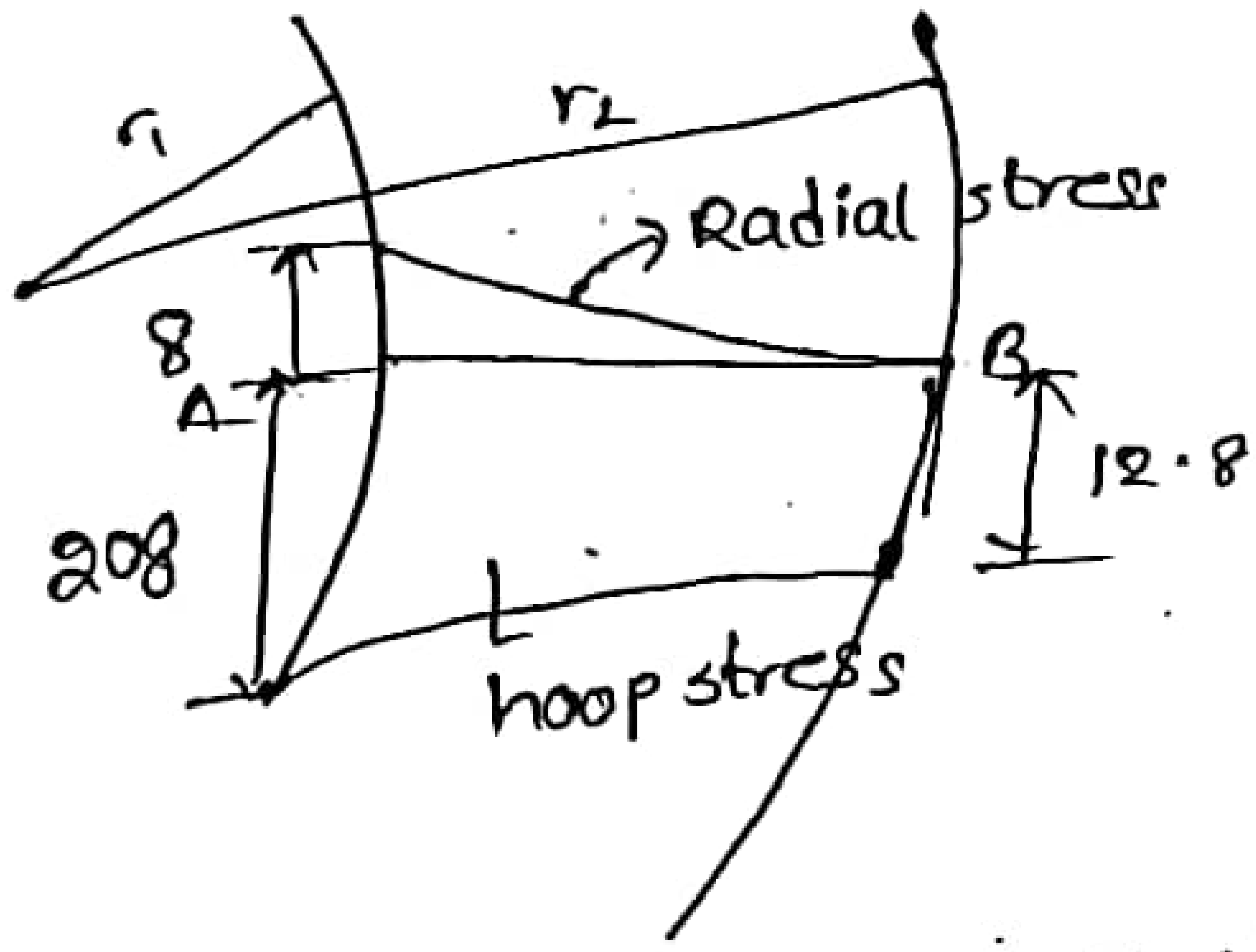
hoop stress (σ_x)

$$\sigma_x \text{ at } r_1 = \sigma_{200} = \frac{576000}{200^2} + 6.4$$

$$\sigma_{200} = 20.8 \ N/mm^2$$

$$\sigma_x \text{ at } r_2 = \sigma_{300} = \frac{576000}{300^2} + 6.4$$

$$\sigma_{300} = 12.8 N/mm^2.$$



(Q) Determine the thickness of the metal for a cylindrical shell of a internal diameter 160 mm to with stand and internal pressure a & 8 N/mm². The max hoop stress is 35 N/mm².

Sol:- Given that: ; $t = r_2 - r_1$

$\sigma_x = 35\, N/mm^2$.

$d = 160\, mm \Rightarrow r_1 = 80\, mm$

$P_x = 8\, N/mm^2$

$\Rightarrow P_x = \frac{b}{x^2} - a$; $8 = \frac{b}{80^2} - a$

at $x = 80\,mm$, $\sigma_x = max = 35\, N/mm^2$

$\sigma_x = \frac{b}{x^2} + a$

$35 = \frac{b}{80^2} + a$

$a = 13.5$, $b = 137600$

at $x = r_2$, $P_x = 0$

$0 = \frac{b}{x_2^2} - a$ $\qquad a = b/x_2^2$

$13.5 = \frac{137600}{x_2^2} \rightarrow x_2 = \sqrt{\frac{137600}{13.5}} \Rightarrow r_2 = 101\,mm$

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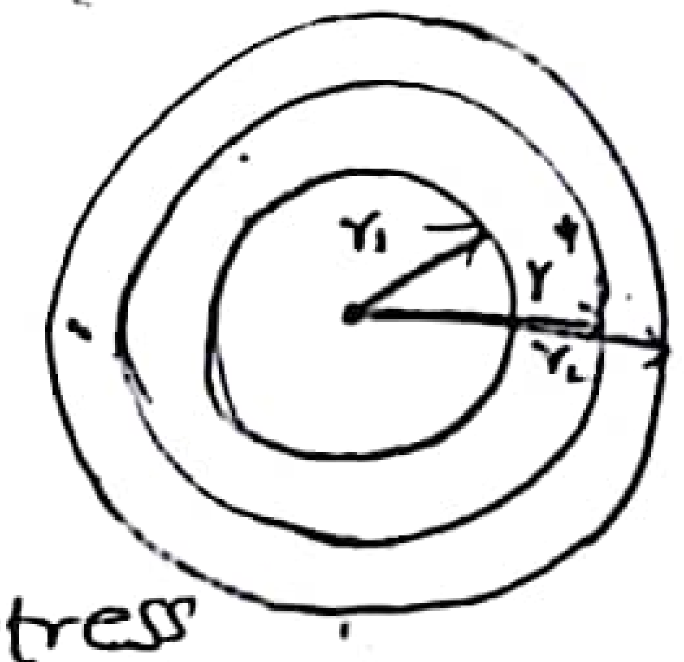
$t = r_2 - r_1 = 10.1 - 80 = 21 mm$

* Stress in compound thick cylinders :-



~~outer cylinder~~ The max hook stress at inner radius is always greating at than internal fluid pressure. Hence the max. fluid pressure inside the cylinder is limited to the; hoops stress at inner radius reaches to permisable value.

In the case of cylinder which have to carry high internal fluid pressure some methods of reducing hoops stress have to divied outer cylinder.

for this purpose we can use compound thick cylinder. Let us assume. Due to shrinkage the inner cylinder will be put in intial compression and outer cylinder be initial tension

$$\sigma_x = \frac{b_2}{x^2} + a_2 .$$

If the cylinder is subjected to internal and outer cylinder will be subjected to hoops tensile stress. The net effective stress due to shrinkage and internal fluid pressure is to make the resultant stresses uniform

Let us assume r_2 is the outer radius of cylinder and r_1 is iner cyclinder and r^* radius of Junction of cylinder

→ Due to Shrikage the inner cylinder under compression Outer cylinder under tension. Assume, p^* radius stress at junction of cyclinder

Apply the lami's equation

$P_x = \frac{b_1}{x^2} - a_1$ ----(6) ; $\sigma_x = \frac{b_1}{x^2} + a$ ----(5)

A & B are k can be find out by following boundary conditions.

Outer cylinder

at $x = r_2$; $P_x = 0$

at $x = r^*$; $P_x = p^*$

∴ By using boundaries condition we can calculated

$P_x = \frac{b_1}{x^2} - a_1$, $\sigma_x = \frac{b_1}{x^2} + a_1$

Inner cylinder

Apply the Lami's equation

at $x = r^*$, $P_x = p^*$; at $x = r_1$; $P_x = 0$

from (5) & (6)

$P_x = \frac{b_2}{x^2} - a_2$; $\sigma_x = \frac{b_2}{x^2} + a_2$

∴ By using boundary condition a_2, b_2 are k's.

By sub a_2 b_2 in σ_x, P_x we can hoops stress and radical stress.

Cylinder subjected to internal pressure:-

when the fluid under pressure is admitted into the compound cylinder the hoops stress are set in the compound cylinder to find the stress the inner & outer cylinder combined and treated as thick cylinder let p is the internal fluid pressure apply the lami's theorem

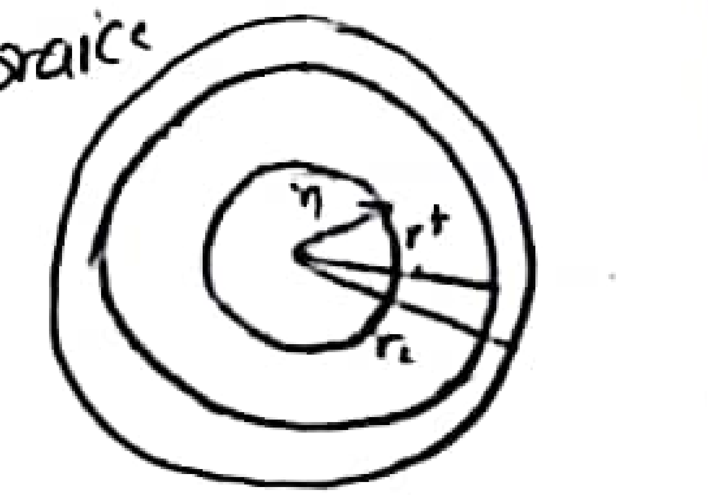
$\sigma_x = \frac{B}{x^2} + A$; $P_x = \frac{B}{x^2} - A$

at $x = r_2$, $P_x = 0$

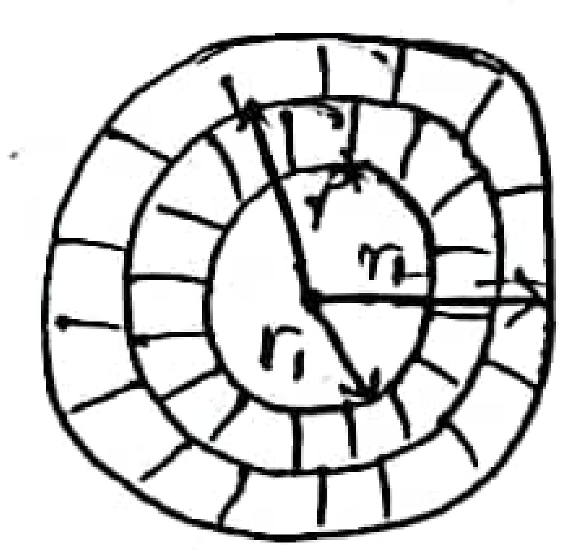
$x = r$, $P_x = P$

A, B are the constants of combined thick cylider which can be calculated by boundary conditions

The resultant hoops stress is the algebraic sum of hoops due to shrikage and hoops due to internal fluid pressure



(Q) A compound cylinder is made by a shrinkage a cylinder of external dia 300mm and internal dia 250mm over a cylinder of external dia of 50mm and internal dia of 200. The radial pressure at junction after shrinkage 8 N/mm². Find the final stress set up across the section when the compound cylinder subjected to an internal pressure of 84.5 N/m².



Sol:- Given that

$d_2 = 300mm$; $r_2 = 150mm$

$d_1 = 200mm$; $r_1 = 100mm$

$d^* = 250mm$; $r^* = 125mm$

$p^* = 8N/mm^2$; $P = 84.5N/mm^2$

Fluid is not under pressure (only shrinkage)

Radial stress $P_x = \frac{b_1}{x^2} - a_1$ — ①

hoop stress $\sigma_x = \frac{b_1}{x^2} + a_1$ — ②

a_1 & b_1 can be calculated by using Boundary conditions

at $x = r_2 = 150mm$, $P_x = 0$ sub ①

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$$0 = \frac{b_1}{150^2} - a_1 \Rightarrow \left[a_1 = \frac{b_1}{150^2}\right]$$

crt $x = r^* = 125\,mm$, $P_x = p^* = 8\,N/mm^2$ sub ①

$$8 = \frac{b_1}{125^2} - a_1 \; ; \; 8 = \frac{b_1}{125^2} - \frac{b_1}{150^2}.$$

$$8 = b_1\left[\frac{1}{125^2} - \frac{1}{150^2}\right]$$

$$8 = b_1\left[1.95\times10^{-5}\right]$$

$$b_1 = \frac{8}{1.95\times10^{-5}} \Rightarrow b_1 = 409090.90$$

$$a_1 = \frac{b_1}{150^2} \Rightarrow a_1 = \frac{b_1}{150^2}$$

$$a_1 = \frac{409090.90}{150^2} = 18.18$$

* hoopstress at outer diameter of outer cylinder

$$\sigma_{150} = \frac{b_1}{x^2} + a_1 = \frac{409090.90}{150^2} + 18.18 = 36$$

hoopstress and Inner parmeter of outer cylinder (at Junction)

$$\sigma_{125} = \frac{b_1}{x^2} + a_1 = \frac{409090.90}{125^2} + 18.18 = 44.36.$$

for Inner cylinder:-

Radical stress $P_x = \frac{b_2}{x^2} - a_2$ —③

hoop stress $\sigma_x = \frac{b_2}{x_2} + a_2$ —④

$a_2 b_2$ can be find at by using Bandary conditions

at $x = r^* = 125\,mm$, $P_x = p^* = 8\,N/mm^2$ sub ③

$$8 = \frac{b_2}{125^2} - a_2 \longrightarrow ⑤$$

$$0 = \frac{b_2}{100^2} - a_1, \Rightarrow a_1 = \frac{b_2}{100^2} \longrightarrow \text{sub in } (?)$$

$$[illegible] = \frac{b_2}{125^2} - \frac{b_2}{100^2}$$

$$[illegible] = b_2\left[\frac{1}{125^2} - \frac{1}{150^2}\right]$$

$$b_2 = -222222.22 \qquad a_1 = \frac{-222222.22}{100^2} = -22.22$$

hoopstress σ = at outer diameter of Inner cylinder (or) r_2

$$\sigma_{125} = \frac{b_2}{r^2} + a_1 = \frac{-222222.22}{125^2} - 22.22 = -36.44 \text{ N/mm}^2$$

hoopstress and Inner diameter of Inner cylinder r_1

$$\sigma_{100} = \frac{b_2}{r^2} + a_1 = \frac{-222222.22}{100^2} - 22.22$$

$[illegible]_{100} = -44.44$ N/mm² (comp).

The radial stress $P_x = \frac{B}{x^2} - A$ —(6)

hoop stress $\sigma_x = \frac{B}{x^2} + A$ —(7)

at $x = r_2 = 150$ mm, $P_x = 0$ sub (6)

$$0 = \frac{B}{150^2} - A \Rightarrow A = \frac{B}{150^2}$$

at $x = r_1 = 100$mm, $P_x = 84.5$ N/mm²

$$84.5 = \frac{B}{x^2} - \frac{B}{150^2} \qquad = 84.5 = \frac{B}{100^2} - \frac{B}{150^2}$$

$$B = 1521000$$

$$A = \frac{1521000}{150^2} = 67.6$$

hoopstress :-

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6 = 135.2 \text{ N/mm}^2$$

$$\sigma_{125} = \frac{1521000}{125^2} + 67.6 = 164.94\, N/mm^2$$

$$\sigma_{100} = \frac{1521000}{100^2} + 67.6 = 219.7\, N/mm^2$$

Net hoop stress

for outer cylinder

$\sigma_{150} = \sigma_{150}$ of shimkage $+ \sigma_{150}$ of fluid pressure

$= 36.36 + 135.2$

$\sigma_{150} = 171.56\, N/mm^2$

$\sigma_{125} = 44.36 + 164.94$

$\sigma_{125} = 209.3\, N/mm^2$

for Inner cylinder

$\sigma_{125} = -36.44 + 164.94$

$\sigma_{125} = 128.5\, N/mm^2$

$\sigma_{100} = -44.44 + 219.7$

$\sigma_{100} = 175.26\, N/mm^2$

* Initial difference in radii at a junction of compound cylinder for shrinkage:-

→ By shrinkage the outer cylinder over inner cylinder, some compressive. stress Produced in inner cylinder.

→ In order to shrink the outer cylinde over inner cylinder, The Inner dia of the outer cylinder should be less than the outer dia of the inner cylinder

→ Now the outer cylinder is heated and inner cylinder is inserted in. to it.

→ ~~After~~ the Cooling the outer cylinder strings over the inner cylinder

→ By this the inner cylinder is put in the compression, outer cylinder is put in the tension

→ ~~After~~ shrinkage the occur radius of inner cylinder decreases and ~~the~~ inner radius of outer cylinder increases

→ Let r_2 is the outer radius of outer cylinder, r_1 is the inner radius of inner cylinder, r^* is the radius at junction after shrinkage, p^* radial pressure at the junction before shrinkage (r_1 ~~...~~ & r_2 ~~...~~) (The inner radius of outer cylinder is less than the r^*. And the outer radius of inner cylinder is greater than the r^*

→ Apply the Lami's eq for both inner and outer cylinder

$$P_x = \frac{b}{x^2} - a \quad \text{—①} \; ; \quad \sigma_x = \frac{b}{x^2} + a \quad \text{—②}$$

for outer cylider $= a_1$ & b_1 ; for inner cylinder $= a_2$ & b_2

→ The radial pressure P_x is same for outer and inner cylinder

$$(P_x)_{\text{outer cylinder}} = (P_x)_{\text{inner cylinder}}$$

$$\frac{b_1}{(r^*)^2} - a_1 = \frac{b_2}{(r^*)^2} - a_2 \quad \text{—①}$$

$$\frac{b_1 - b_2}{(r^*)^2} = a_1 - a_2$$

$$\boxed{b_1 - b_2 = (r^*)^2 (a_1 - a_2)} \quad \text{—②}$$

hoop strain or circumferential strain

$$e_h = \frac{\sigma_x}{E} - \mu\left(\frac{-P_x}{E}\right) \quad \left[e_h = \frac{\sigma_x}{E} + \frac{\mu P_x}{E}\right] \text{—③}$$

But hoop strain (e) & circumference = change in circumference / initial circumference

$$e_h = \frac{\text{final C.F} - \text{Initial C.F}}{\text{Initial C.F}}$$

$$e_h = \frac{2\pi(r+d_r) - 2\pi r}{2\pi r}, \quad e_h = \frac{\cancel{2\pi r} + 2\pi dr - \cancel{2\pi r}}{2\pi r}$$

$$e_h = \frac{dr}{r} \quad \text{— ④}$$

from eq ③ & ④

$$\frac{dr}{r} = \frac{\sigma_1}{E} + \frac{\mu P_1}{E} \quad \Rightarrow \frac{dr}{r} =$$

$$dr = r\left[\frac{\sigma_1}{E} + \frac{\mu P_1}{E}\right] \quad \text{— ⑤}$$

On shrinkage the extortion (increases) inner radius of the outer cylinder. and contraction (decreases) outer radius of the inner cylinder.

→ At the junction inner radius of outer cylinder increased

∴ the increasing in inner radius of outer cylinder (dr)

$$dr = r^*\left[\frac{\sigma_x}{E} + \frac{\mu P_x}{E}\right] \quad \text{— ⑥}$$

For outer cylinder $P_x = \frac{b_1}{(r^*)^2} - a_1$; $\sigma_x = \frac{b_1}{(r^*)^2} + a_1$

Sub these two in eqn ⑥.

$$= r^*\left[\frac{\frac{b_1}{(r^*)^2} + a_1}{E} + \frac{\mu\left[\frac{b_1}{(r^*)^2} - a_1\right]}{E}\right] \quad \text{— ⑦}$$

→ At the junction outer radius of inner cylinder is decreasing

$\therefore$ The decreasing of outer radius of inner cylinder =

$$-r^*\left[\frac{\sigma_x}{E}+\frac{\mu P_x}{E}\right] \quad —⑧$$

For inner cylinder $P_x = \frac{b_2}{(r^*)^2} - a_2$ $\quad \sigma_x = \frac{b_2}{(r^*)^2} + a_2$

$$\text{Now} = -r^*\left[\frac{1}{E}\left[\frac{b_2}{(r^*)^2}+a_2\right]+\frac{\mu}{E}\left[\frac{b_2}{(r^*)^2}-a_2\right]\right] \quad —⑨$$

But At junction the original difference b/w the outer radius of inner cylinder and inner radius of outer cylinder = increasing in Inner radius of outer cylinder + decreasing in outer radius of inner cylinder.

$$= r^*\left[\frac{1}{E}\left[\frac{b_1}{(r^*)^2}+a_1\right]+\frac{\mu}{E}\left[\frac{b_1}{(r^*)^2}-a_1\right]\right] +$$

$$-r^*\left[\frac{1}{E}\left[\frac{b_2}{(r^*)^2}+a_2\right]+\frac{\mu}{E}\left[\frac{b_2}{(r^*)^2}-a_2\right]\right]$$

$$= \frac{r^*}{E}\left[\frac{b_1}{(r^*)^2}+a_1 - \frac{b_2}{(r^*)^2}+a_2\right]+\frac{r^*\mu}{E}\left[\underbrace{\left(\frac{b_1}{(r^*)^2}-a_1\right)\cdot\left(\frac{b_2}{(r^*)^2}-a_2\right)}_{\text{from eq ① these two are equal}}\right]$$

$$\frac{b_1}{(r^*)^2} - a_1 = \frac{b_2}{(r^*)^2} - a_2$$

$$= \frac{r^*}{E}\left(\frac{b_1}{(r^*)^2}+a_1-\frac{b_2}{(r^*)^2}-a_2\right)$$

$$= \frac{r^*}{E}\cdot\left(\frac{b_1-b_2}{(r^*)^2}+(a_1-a_2)\right)$$

$$= \frac{r^*}{E}\left((a_1 - a_2) + (a_1 - a_2)\right)$$

$$= \frac{r^*}{E} 2(a_1 - a_2)$$

$$\boxed{= \frac{2r^*}{E}(a_1 - a_2)}$$

$-a_2)]$

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MODULE-IV

COLUMNS AND STRUTS

UNIT- 2. Columns and Struts

A member of a structure or a bar which carries an axial compressive load is called as strut. If struct is vertical i.e ; 90° to the horizontal is known as pillar column.

→ A member in any position other than vertical subjected to compressive load is called as column.

→ The difference b/w strut and column is struct may have its one or both ends fixed rigidly / hinged / pin jointed. while the column will have both ends fixed rigidly.

Ex:- Connecting rods, piston rods, side links in foreign machine.

Reasons for failures of struts:-

→ By pure compression.

→ Buckling / crippling.

→ By combination of both compression and buckling.

Buckling / Buckling load :-

The max limiting load at which the column tends to have lateral replacement / buckle is called buckling load.

The buckling load about an axis having min. radius of variation / least moment of inertia.

Radius of gyration (K):-

The entire area is concentrated at a point to distance of this point from given axis of reference is called as radius of gyration.

$$k = \sqrt{\frac{I}{A}}$$

where, I = moment of inertia

A = Area of column.

Slenderness ratio (λ):- It is the ratio of actual length of column to the min/ least radius of gyration.

$$\lambda = l/K_{min.}$$

Buckling factor:- It is the ratio b/w equivalent length of the column to min. radius of gyration.

$$\text{Buckling factor} = \frac{l_e}{K_{min}}$$

Safe load:- It is the load on the column to which it is actually subjected to well below buckling load.

$$\text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety.}}$$

Classification of columns:-

Depending upon slenderness ratio / length of dia ratio columns can be divided into 3 types.

1. **short column**:-

columns which have length less than 8 times of the diameter or slenderness ratio is less than 32 are called as short columns.

→ short columns are always subjected to direct compressive stress and buckling stress are very small.

2. **Medium size column**:-

The column which have their length varying from 8 times to 30 times to the diameter ($8 < l/d < 30$) or slenderness ratio lying b/w to 32 to 120 are called as medium size columns / Intermediate columns.

→ In this column both buckling as well as direct compressive stress have significant values.

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3. **Long column :-**

The columns having more than 30 times their dia ($l/d > 30$) or slenderness ratio more than 120 are known long columns.

Equivalent length :- The distance b/w adjacent point of inflection is called equivalent length. Effective length are simple length of column.

→ At a point of inflection is formed at every column is free to rotate.

Strength of column :- The strength of a column depends upon a slenderness ratio. If slenderness ratio increases the compressive strength of column decreases as the tendency to buckle is increased. The strength of column depends upon end conditions also.

End conditions of a column :-

There are 4 conditions : They are :-

1. Both ends are hinged / pin jointed. 

2. One end is fixed, othere is free. 

3. Both ends are fixed. 

4. One end is fixed and other end is hinged or pin jointed. 

Euler's theory :- According to Euler's theory, the failures of long column are only by buckling stress and direct compressive stress are very negligible due to more length of column compared with dia.

Assumptions made in Euler's theory :-

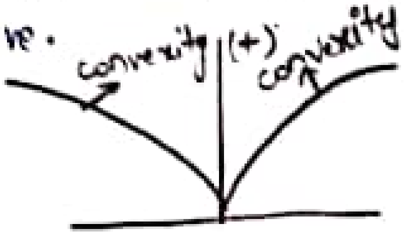
→ The column is initially straight and uniform in lateral dimensions.

→ The material of the column is gently homogeneous or isotropic.

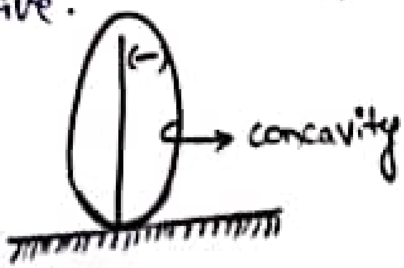
- The compressive load is exactly axial and it passes through centroid of column section.
- The self weight of the column is neglected.
- limit of proportionality is not exceed.
- The columns fails by buckling only / buckling alone.
- pin joints / hinged joints are frictionless and fixed ends are perfectly rigid.

Sign conventions for bending moment :-

1. A bm which bends the column so as to present convexity towards the initial central line of the members will be taken as positive.



2. A BM which bends the columns present the concavity towards the initial center line of the members will be taken as negative.



Euler's formula (P_c) :-

$$P_c = \frac{\pi^2 EI}{l_e^2}.$$

where P_c = crippling load / buckling load.

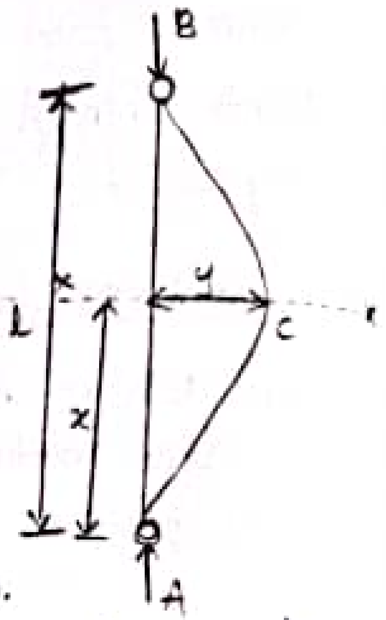
E = young's modulus.

I = Moment of inertia.

l_e = effective length.

Euler's formula is used for calculating the critical load for a column / strut.

*** Expression for crippling load when both the ends of the column are hinged or pin jointed:-**



The load at which column just buckles / bends is called crippling load.

Consider a column AB of length 'l' and uniform cross-sectional area, hinged at both the ends A and B. let P be the crippling load at which the column has just buckled. Due to crippling load the column will defeat into the curved form ACB as shown.

Consider any section x-x at a distance 'x' from end A let y = deflection at the section x-x.

Now, BM @ x-x, $M = -Py \longrightarrow (1)$

But WKT, $BM = EI\frac{d^2y}{dx^2} \longrightarrow (2)$

(1) = (2)

$$EI\frac{d^2y}{dx^2} = -Py$$

$$EI\frac{d^2y}{dx^2} + Py = 0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

The solution for above differential equation is,

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{P/EI}\right) \longrightarrow (3)$$

Now, At end A, $x=0$ and $y=0$

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put above boundary conditions in ②.

$$0 = C_1 \cos\left(0\sqrt{P/EI}\right) + C_2 \sin\left(0\sqrt{P/EI}\right)$$

$$0 = C_1 + 0$$

$$C_1 = 0$$

Now, At $x = l$ and $y = 0$.

put above boundary conditions in ③

$$0 = 0\cos\left(l\sqrt{P/EI}\right) + C_2 \sin\left(l\sqrt{P/EI}\right)$$

$$0 = 0\cos\left(l\sqrt{P/EI}\right) \cancel{+ C_2 \sin}$$

$$C_2 = 0 \quad \text{or} \quad \sin\left(l\sqrt{P/EI}\right) = 0$$

But $C_2 \neq 0$

so, $\sin\left(l\sqrt{P/EI}\right) = 0 \rightarrow l\sqrt{\frac{P}{EI}} = \sin^{-1}(0)$

$$l\sqrt{\frac{P}{EI}} = 0, \pi$$

$$l\sqrt{\frac{P}{EI}} = \pi \quad \text{(squaring)}$$

$$l^2\left(P/EI\right) = \pi^2$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

Now, $P_c = \dfrac{\pi^2 EI}{l_e^2}$ comparing both we get

$$\boxed{l_e = l}$$

* <u>Expression for crippling load of a column when one end is fixed and other end is free</u> :-

Now, BM @ x-x, $M = P(a-y) \longrightarrow ①$

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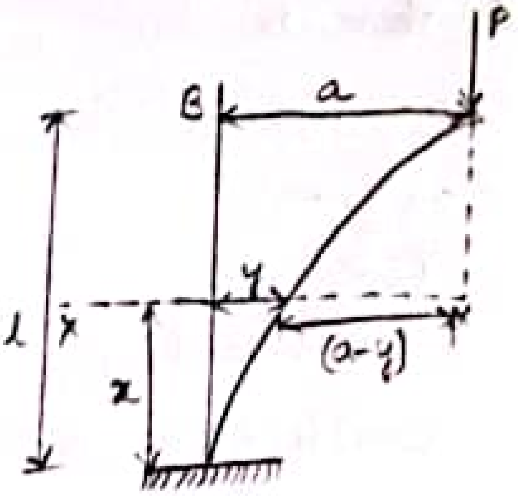
But w.k.t.

BM $M = EI\frac{d^2y}{dx^2} \longrightarrow (2)$



$(1) = (2)$

$$EI\frac{d^2y}{dx^2} = P(a-y)$$

$$EI\frac{d^2y}{dx^2} + Py = Pa$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{Pa}{EI}$$

Now, The solution for above DE is

$$y = c_1\cos\left(x\sqrt{P/EI}\right) + c_2\sin\left(x\sqrt{P/EI}\right) + a \longrightarrow (3)$$

Now, At end A $x=0, y=0$

now, put above B.C in (3)

$$0 = c_1\cos\left(0\sqrt{P/EI}\right) + c_2\sin\left(0\sqrt{P/EI}\right) + a$$

$$0 = c_1 + a$$

$$c_1 = -a$$

Now, Diff eqn (3) w.r.to x

Now, At end A, $x=0, \frac{dy}{dx}=0$

put above BC in (4) we get.

$$0 = a\sin\left(0\sqrt{P/EI}\right)\sqrt{P/EI} + c_2\cos\left(0\sqrt{P/EI}\right)\sqrt{P/EI}.$$

$$0 = c_2\sqrt{P/EI}$$

$\therefore c_2 = 0$

Now, At end B $x=l, y=a$

put above B.C in (3)

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$$a = -a\cos\left(l\sqrt{P/EI}\right) + 0\sin\left(l\sqrt{P/EI}\right) + a$$

$$\alpha = -a\cos\left(l\sqrt{P/EI}\right) + \alpha$$

$$0 = -a\cos\left(l\sqrt{P/EI}\right)$$

$$\cos\left(l\sqrt{P/EI}\right) = 0$$

$$l\sqrt{P/EI} = \cos^{-1}(0)$$

$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \frac{3\pi}{2}, ----$$

$$l\sqrt{P/EI} = \frac{\pi}{2}.$$

Now, S.O.B.S

$$l^2\left(\frac{P}{EI}\right) = \frac{\pi^2}{4}$$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

~~Now S.O.B.S~~ $P = \frac{\pi^2 EI}{(2l)^2}$

Now, Euler's $P_e = \frac{\pi^2 EI}{l_e^2}$

Comparing Both we get

$$P = P_e$$

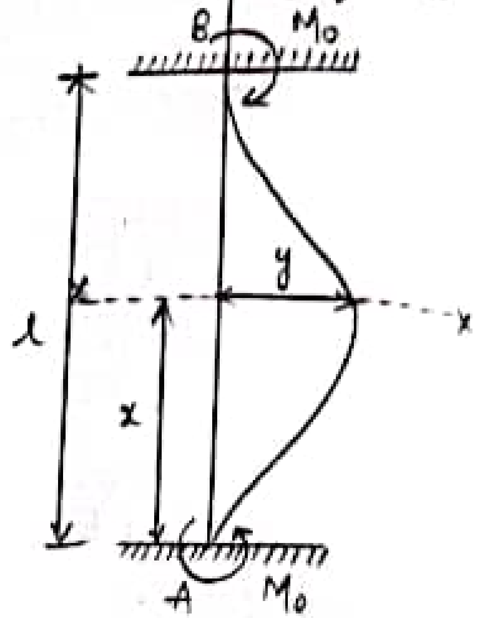
$$\frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 EI}{l_e^2}$$

$$l_e^2 = (2l)^2$$

$$\boxed{l_e = 2L}$$

* Expression for crippling load by a column when both the ends are fixed :-

Due to fixed ends A and B the moments are formed @ A & B i.e, fixed ends (M_0)



Now,

BM @ x-x, $M = M_0 - Py \longrightarrow ①$

But w.k.t

BM, $M = EI\dfrac{d^2y}{dx^2} \longrightarrow ②$

① = ②

$$EI\frac{d^2y}{dx^2} = M_0 - Py$$

$$EI\frac{d^2y}{dx^2} + Py = M_0$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{EI}$$

Now, multiply and divide by P on right side.

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI} \times \frac{M_0}{P}$$

Now, solution for above D.E is

$$y = C_1\cos\left(x\sqrt{P/EI}\right) + C_2\sin\left(x\sqrt{P/EI}\right) + \frac{M_0}{P} \longrightarrow ③$$

At end A, $x=0, y=0$

put above B.C in ③

$$0 = C_1\cos\left(0\sqrt{P/EI}\right) + C_2\sin\left(0\sqrt{P/EI}\right) + \frac{M_0}{P}$$

$$0 = C_1 + \frac{M_0}{P}$$

$$C_1 = \frac{-M_0}{P}$$

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Now, differentiating eqn ③ w.r.to x.

$$\frac{dy}{dx} = -C_1 \sin\left(x\sqrt{P/EI}\right)\sqrt{P/EI} + C_2\cos\left(x\sqrt{P/EI}\right)\sqrt{P/EI} \rightarrow ④$$

Now, At end A, $x=0$, $\frac{dy}{dx}=0$

put above B.C in ④

$$0 = \frac{M_0}{P}\sin\left(0\sqrt{P/EI}\right)\sqrt{P/EI} + C_2\cos\left(0\sqrt{P/EI}\right)\sqrt{P/EI}$$

$$0 = C_2\sqrt{P/EI}$$

$$C_2 = 0$$

Now, At end B, $x=l$, $y=0$

put above B.C in ③

$$0 = -\frac{M_0}{P}\cos\left(l\sqrt{P/EI}\right) + 0\sin\left[l\sqrt{P/EI}\right] + \frac{M_0}{P}$$

$$0 = -\frac{M_0}{P}\cos\left(l\sqrt{P/EI}\right) + \frac{M_0}{P}$$

$$\frac{M_0}{P} = \frac{M_0}{P}\cos\left(l\sqrt{P/EI}\right)$$

$$\cos\left(l\sqrt{P/EI}\right) = 1$$

$$l\left(\sqrt{P/EI}\right) = \cos^{-1}(1)$$

$$l\left(\sqrt{P/EI}\right) = 0, 2\pi \ldots$$

$$l\left(\sqrt{P/EI}\right) = 2\pi.$$

squaring on both sides.

$$l^2\left(\frac{P}{EI}\right) = 4\pi^2$$

$$P = \frac{4\pi^2 EI}{l^2}$$

$$P = \frac{\pi^2 EI}{l^2/4}$$

$$\boxed{P = \frac{\pi^2 EI}{(l/2)^2}}$$

Now, Eulers $P_c = \frac{\pi^2 EI}{l_e^2}$

comparing both we get

$$P = P_c$$

$$\frac{\pi^2 EI}{(l/2)^2} = \frac{\pi^2 EI}{l_e^2}$$

$$l_e^2 = (l/2)^2$$

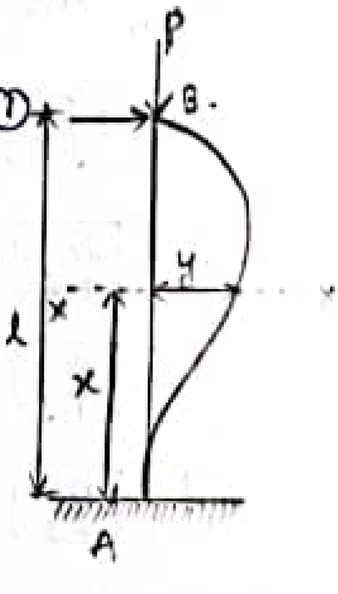
$$\boxed{l_e = l/2}$$

Expression for crippling load of a column when one end is fixed and other end is hinged:-

Now, BM @ x-x, $M = -Py + H(l-x) \rightarrow (1)$

But w.k.t

B.M, $M = EI\frac{d^2y}{dx^2} \longrightarrow (2)$



Now (1) = (2)

$$EI\frac{d^2y}{dx^2} = H(l-x) - Py$$

$$EI\frac{d^2y}{dx^2} + Py = H(l-x)$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{H(l-x)}{EI}$$

Multiply and divide with P on right side.

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI} \times \frac{H(l-x)}{P}$$

Now, solution for above D.E is

$$y = c_1 \cos\left[x\sqrt{P/EI}\right] + c_2 \sin\left[x\sqrt{P/EI}\right] + \frac{H(l-x)}{P} \longrightarrow (3)$$

Now At end A, $x=0, y=0$

$$0 = c_1 \cdot \cos\left[0\sqrt{P/EI}\right] + c_2 \sin\left(0\sqrt{P/EI}\right] + \frac{H(l-0)}{P}$$

$$0 = c_1 + \frac{Hl}{P}$$

$$c_1 = -\frac{Hl}{P}$$

Now, Diff (3) w.r.to x.

$$\frac{dy}{dx} = -c_1 \sin\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + c_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} - \frac{H}{P} \longrightarrow (4)$$

Now, At end A, $x=0$, $\frac{dy}{dx}=0$

$$0 = \frac{Hl}{P} \sin\left(0\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + c_2 \cos\left(0\sqrt{P/EI}\right)\sqrt{P/EI} - \frac{H}{P}$$

$$0 = c_2\sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$\frac{H}{P} = c_2\sqrt{\frac{P}{EI}}$$

$$c_2 = \frac{H}{P}\sqrt{\frac{EI}{P}}$$

Now, At end B.

$x = l, y = 0$

put above B.C in (3)

$$0 = -\frac{Hl}{P} \cos\left(l\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(l\sqrt{\frac{P}{EI}}\right) + \frac{H(l-l)}{P}$$

$$\frac{\cancel{l}}{\cancel{P}}\cos\left[l\sqrt{\frac{P}{EI}}\right] = \frac{\cancel{l}}{\cancel{P}}\sqrt{\frac{EI}{P}}\sin\left(l\sqrt{\frac{P}{EI}}\right]$$

$$l\cos\left(l\sqrt{\frac{P}{EI}}\right] = \sqrt{\frac{EI}{P}}\sin\left(l\sqrt{\frac{P}{EI}}\right]$$

$$\frac{\sin\left(l\sqrt{\frac{P}{EI}}\right)}{\cos\left(l\sqrt{\frac{P}{EI}}\right)} = l\sqrt{\frac{P}{EI}}$$

$$\mathrm{Tan}\left(l\sqrt{\frac{P}{EI}}\right] = l\sqrt{\frac{P}{EI}}$$

Note :- If $\tan\theta = \theta$, then $\theta = 4.5$ radians

So, $l\sqrt{\frac{P}{EI}} = 4.5$

Squaring on both sides

$$l^2\left(\frac{P}{EI}\right] = 4.5^2$$

$$l^2\left[\frac{P}{EI}\right] = 20.25$$

$$P = \frac{20.25\,EI}{l^2}$$

$$P = \frac{2\pi^2 EI}{l^2}$$

$$P = \frac{\pi^2 EI}{l^2/2}$$

$$\boxed{P = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2}}$$

Now, Euler's $P_c = \frac{\pi^2 EI}{l_e^2}$

comparing both we get

$P = P_c$

$$\frac{\pi^2 EI}{(l/\sqrt{2})^2} = \frac{\pi^2 EI}{l_e^2}$$

$$l_e^2 = (l/\sqrt{2})^2$$

$$\boxed{l_e = l/\sqrt{2}}$$

* Crippling stress in terms of effective length and radius of gyration :-

The MOI 'I' can be expressed in terms of radius of gyration and it is given as

$$K = \sqrt{\frac{I}{A}}$$

$$I = AK^2$$

Now, $P_c = \frac{\pi^2 EI}{l_e^2}$

$$P_c = \frac{\pi^2 EAK^2}{l_e^2}$$

$$P_c = \frac{\pi^2 EA}{\left(\frac{l_e}{K}\right)^2}$$

Now, crippling stress $= \frac{P_c}{A}$

$$= \frac{\pi^2 EA}{\left(\frac{l_e}{K}\right)^2}$$

$$= \frac{\frac{\pi^2 EA}{\left(\frac{l_e}{K}\right)^2}}{A}$$

$$\boxed{\text{Crippling stress} = \frac{\pi^2 E}{\left(\frac{l_e}{K}\right)^2}}$$

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- A solid long bar of 3cm length and 5cm diameter is used as struck with both ends hinged. Determine the crippling load by taking $E = 2\times10^5\ N/mm^2$. Also determine the crippling load when i) one end of the strut is fixed and other end is free.
ii) Both ends of the strut are fixed.
iii) one end is fixed and other is hinged.

Given:-

$l = 3m = 3000\,mm$

$D = 5\,cm = 50\,mm$

$E = 2\times10^5\ N/mm^2$

i) w.k.t

$$P = \frac{\pi^2 EI}{l^2}$$

But

$$I = \frac{\pi}{64}D^4 = \frac{\pi}{64}\times 50^4 = 306796\ mm^4$$

Now,

$$P = \frac{\pi^2\times2\times10^5\times306796.15}{(3000)^2} = 67287.9\,N$$

$$= 67.28\,kN$$

ii) w.k.t

$$P = \frac{\pi^2 EI}{(2l)^2}$$

$$P = \frac{\pi^2\times2\times10^5\times306796.15}{(2\times3000)^2}$$

$$P = 16821.98\,N = 16.82\,kN$$

iii) w.k.t $P = \dfrac{\pi^2 EI}{(l/2)^2}$

$$P = \frac{\pi^2\times2\times10^5\times306796.15}{\left(\frac{3000}{2}\right)^2}$$

$p = 269151.70\ N$

$p = 269.15\ KN$

iv) w.k.t

$$p = \frac{\pi^2 E I}{\left(\frac{l}{\sqrt{2}}\right)^2}$$

$$p = \frac{\pi^2 \times 2\times10^5 \times 306796.15}{\left(\frac{3000}{\sqrt{2}}\right)^2}$$

$P = 134575.85\ N$

$p = 134.57\ KN$

A column of timber section 15cm x 20cm is 6m long with both ends being fixed. If E = 17.5 KN/mm², find the crippling load and also the safe load of the column. If factor of safety is 3.

Given:

$l = 6m = 6000mm$

$b = 15cm = 150mm$

$d = 20cm = 200mm$

$E = 17.5\ KN/mm^2$

$E = 17.5\times10^3\ N/mm^2$

f.o.s = 0

w.k.t

$$I_{xx} = \frac{bd^3}{12} = \frac{150\times200^3}{12} = 1\times10^8\ mm^4$$

$$I_{yy} = \frac{db^3}{12} = \frac{200\times150^3}{12} = 56250000\ mm^4$$

Now, I = least value of I_{xx}, I_{yy}.

$I = 56250000\ mm^4$.

Now, $p = \frac{\pi^2 EI}{(l/2)^2}$

$$p = \frac{\pi^2 \times 17.5 \times 10^3 \times 56250000}{\left(\frac{6000}{2}\right)^2}$$

$p = 1079437.9\ N$

$p = 1079.43\ KN$

w.k.t

$$\text{Safe load} = \frac{\text{buckling load / crippling load}}{\text{F.O.S}}$$

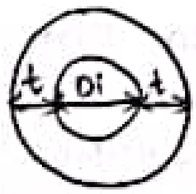
$$= \frac{1079.48}{3}$$

Safe load $= 359.82\ KN$

A hollow mild steel tube of 4cm internal diameter and 5mm thick ~~am~~ is used as strut with both ends being hinged. Find the crippling load and safe load by taking F.O.S = 3 and $E = 2 \times 10^5\ N/mm^2$

Given:- $D_i = 4cm = 40mm$

$t = 5\ mm$



$D_o = D_i + 2t$

$D_o = 40 + 2 \times 5 = 50mm$

$E = 2 \times 10^5\ N/mm^2$

F.O.S = 3

Now, $I = \frac{\pi}{64}\left[D_o^4 - D_i^4\right]$

$= \frac{\pi}{64}\left[50^4 - 40^4\right]$

$I = 181132.45\ mm^4$

Now, w.k.t $p = \frac{\pi^2 EI}{l^2}$

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$$P = \frac{\pi^2 \times 2\times10^5 \times 181132.45}{6000^2}$$

$$= 9931.69 \text{ N}$$

$$P = 9.9 \text{ KN}$$

w.k.t safe load $= \dfrac{\text{crippling load}}{\text{F.O.S}}$

$$= \frac{9.9}{3}$$

safe load $= 3.3$ KN

A simply supported beam of length 4m is subjected to a uniformly distributed load of 30 kN/mm over the whole span and deflects 15 mm at centre. Determine the crippling load when this beam is used as column with following end corrections.

i) one end is fixed and other end is hinged.

ii) Both ends fixed.

iii) Both ends pinned.

iv) one end is fixed and other end is free.

Soln:- Given:-

$w = 30$ kN/m $= 30$ N/mm

$l = 4$m $= 4000$ mm

$\delta = 15$mm

w.k.t $\delta = \dfrac{5wl^4}{384EI}$

$$15 = \frac{5\times30\times4000^4}{384\times EI}$$

$$EI = \frac{5\times30\times4000^4}{384\times15}$$

$EI = 6.66\times10^{12}$ Nmm^2

i) w.k.t

$$P = \frac{\pi^2 EI}{\left(l/\sqrt{2}\right)^2}$$

$$P = \frac{\pi^2 \times 6.66 \times 10^{12}}{\left(4000/\sqrt{2}\right)^2}$$

$P = 8216445.66$ N

$P = 8216.4$ kN

ii) w.k.t

$$P = \frac{\pi^2 EI}{\left(l/2\right)^2}$$

$$= \frac{\pi^2 \times 6.66 \times 10^{12}}{\left(4000/2\right)^2}$$

$P = 16432891.3$ N

$P = 16432.8$ kN

iii) w.k.t $P = \frac{\pi^2 EI}{l^2}$

$$= \frac{\pi^2 \times 6.66 \times 10^{12}}{(4000)^2}$$

$= 4108222.8$ N

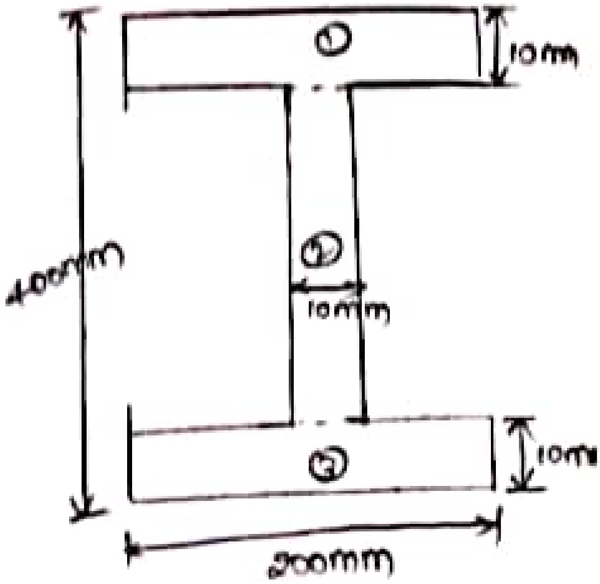
$P = 410822$ kN

iv) w.k.t $P = \frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 \times 6.66 \times 10^{12}}{(2 \times 4000)^2}$

$P = 1027055.70$ N

$P = 1027.05$ kN

Determine the euler's crippling load for an I-section joist 40×40×1 cm and 5m long which is used as a strut with both ends fixed. Take E for the joist as 2.1×10^5 N/mm².



Ans: Given $l = 5m = 5000mm$
$E = 2.1\times10^5 N/mm^2$

Now, $I_{xx} = \frac{b_1d_1^3}{12} + \frac{b_2d_2^3}{12} + \frac{b_3d_3^3}{12}$

$= \frac{200\times10^3}{12} + \frac{10\times380^3}{12} + \frac{200\times10^3}{12}$

$I_{xx} = 4576\times10^4 \ mm^4$

Now, $I_{yy} = \frac{d_1b_1^3}{12} + \frac{d_2b_2^3}{12} + \frac{d_3b_3^3}{12}$

$= \frac{10\times200^3}{12} + \frac{380\times10^3}{12} + \frac{10\times200^3}{12}$

$I_{yy} = 13365\times10^3$

$I_{yy} = 1336.5\times10^4 \ mm^4$

Now, I = least value of I_{xx}, I_{yy}.

$I = 1336.5\times10^4 \ mm^4$

w.k.t $P = \frac{\pi^2 EI}{(l/2)^2}$

$= \frac{\pi^2\times2.1\times10^5\times1336.5\times10^4}{(5000/2)^2}$

$P = 4432080 \ N$

$P = 4432.08 \ kN$

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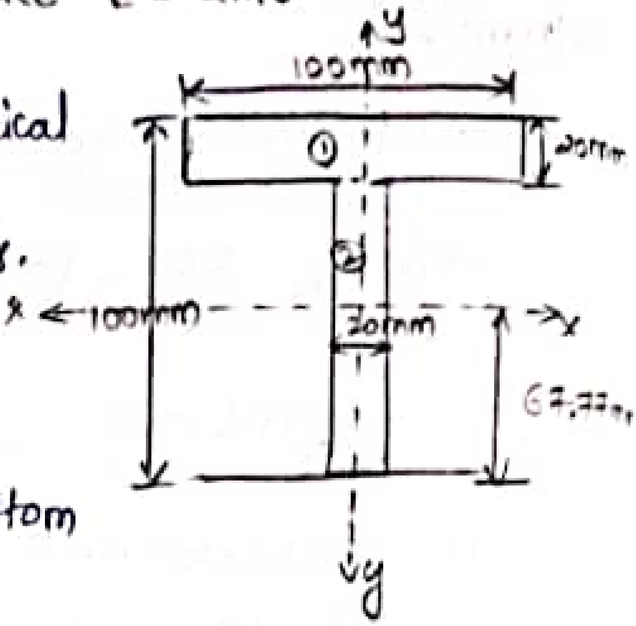
Determine the crippling load for a T-section of dimensions 10x10x2 cm of length 5m when it is used as a strut with both ends being hinged. Take $E = 2\times10^5\ N/mm^2$

Given:

Now



The given T-section is symmetrical about y-y axis.

So the centroid lies along y axis.

Now,
$$a_1 = 100\times20 = 2000mm^2$$
$$a_2 = 80\times20 = 1600\ mm^2$$

Taking reference axis from bottom

So, $y_1 = 80+\frac{20}{2} = 90\ mm$

$$y_2 = \frac{80}{2} = 40\ mm$$

w.k.t
$$\bar{y} = \frac{a_1y_1+a_2y_2}{a_1+a_2}$$
$$= \frac{2000\times90+1600\times40}{2000+1600}$$
$\bar{y} = 67.77$ mm from bottom.

Now, $I_{xx} = \left(\frac{b_1d_1^3}{12}+a_1h_1^2+\frac{b_2d_2^3}{12}+a_2h_2^2\right)$
$$= \frac{100\times20^3}{12}+2000\times(90-67.77)^2+\frac{20\times80^3}{12}+1600\times(67.77-40)^2$$
$$I_{xx} = 3142244\ mm^4$$

$$I_{yy} = \frac{d_1b_1^3}{12}+\frac{d_2b_2^3}{12}+\cancel{\frac{d_3b_3^3}{12}}$$
$$= \frac{20\times100^3}{12}+\frac{80\times20^3}{12}+$$
$$= 172\times10^4\ mm^4$$

$I_{yy} = 1720000 \ mm^4$

Now, I = least value of I_{xx}, I_{yy}.

$I = 1720000 \ mm^4$.

Now,

w.k.t $\quad P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2\times10^5 \times 1720000}{5000^2}$

$P = 135805.75N$

$P = 135.80 \ kN$

* **Limitations of Euler's formula:-**

Crippling stress $= \frac{\pi^2 E}{\left(\frac{l_e}{k}\right)^2}$

Consider a column with both the ends hinged then $l_e = l$

Now, Crippling stress $= \frac{\pi^2 E}{\left(\frac{l_e}{k}\right)^2}$

Consider a whether $\frac{l}{k}$ = slenderness ratio.

If the slenderness ratio is small then crippling stress will be high.

But, for the column material the crippling stress cannot be greater than the crushing stress. Hence the slenderness ratio is less than a certain limit. Euler's formula gives a value of crippling stress, which is greater than crushing stress in the limiting case. we can find the value of $\frac{l}{k}$ for which the crippling stress is equal to crushing stress.

for example, The mild steel column with both the ends are hinged.

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The value of $E = 2.1 \times 10^5 \, N/mm^2$ and the crushing stress $= 330 \, N/mm^2$.

Now, crippling stress = crushing stress.

$$\frac{\pi^2 E}{(l/k)^2} = 330$$

$$\frac{\pi^2 \times 2.1 \times 10^5}{(l/k)^2} = 330$$

$$(l/k)^2 = 6280.65$$

$$l/k = 79.25 \simeq 80$$

$$l/k = 80$$

Hence, if slenderness ratio is small then crippling stress will be high.

But, for the column material the crippling stress cannot be greater than the crushing stress. Hence the slenderness ratio

Rankines formula:-

Eulers formula gives correct results only for the long columns. Rankine establish an emperical formula which is applicable to all columns whether they are short ~~or~~ or long. The emperical formula given by Rankine is known as Rankines formula.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

where,

P = crippling load by Rankine's formula.

P_c = crushing load $\Rightarrow P_c = \sigma_c \cdot A$

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P_E = crippling load by Euler's formula = $\frac{\pi^2 EI}{l_e^2}$

for a given column material the crushing stress σ_c is constant and hence the crushing load P_c will be constant for a given cross-sectional area of the column.

In the above equation P_c is constant and hence the value of P depends upon the value of P_E. for the given column material and given cross sectional area, the value of P_E depends upon the effective length of the column.

<u>Condition</u>:-

1) If the column is short which means the value l_e is small, then the value of P_E will be large and hence the value of $\frac{1}{P_E}$ will be small and negligible as compared to the value of $\frac{1}{P_c}$.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

Neglecting the value of $\frac{1}{P_E}$ and equating, we get.

$$\frac{1}{P} = \frac{1}{P_c}$$

$$\boxed{P = P_c}$$

2) If the column is long, which means the value of l_e is large, then the value of P_E will be small and hence the value of $\frac{1}{P_E}$ will be large when compared to $\frac{1}{P_c}$.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

Neglecting the value of $\frac{1}{P_c}$, we get

$$\frac{1}{P} = \frac{1}{P_E}$$

$$\boxed{P = P_E}$$

Hence the Rankines formula is applicable to both short columns and long columns.

Now,

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

$$\frac{1}{P} = \frac{P_E + P_c}{P_E P_c}$$

$$P = \frac{P_E P_c}{P_E + P_c}$$

$$P = \frac{\frac{P_E P_c}{P_E}}{\frac{P_E}{P_E} + \frac{P_c}{P_E}}$$

$$P = \frac{P_c}{1 + \frac{P_c}{P_E}}$$

$$P = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A}{\frac{\pi^2 EI}{l_e^2}}}$$

Now,

$\frac{\sigma_c}{\pi^2 E}$ is known as Rankines constant (α)

$$P = \frac{\sigma_c A}{1 + \frac{\alpha A l_e^2}{I}}$$

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Now,

$$K = \sqrt{\frac{I}{A}} \Rightarrow K^2 = \frac{I}{A}$$

$$\frac{1}{K^2} = \frac{A}{I}$$

$$P = \frac{\sigma_c A}{1 + \frac{\alpha . l_e^2}{K^2}}$$

$$\boxed{P = \frac{\sigma_c A}{1 + \alpha\left(\frac{l_e}{K}\right)^2}}$$

S.No.	Material	σ_c (N/mm^2)	α
1.	Wrought iron	250	$\frac{1}{900}$
2.	Cast Iron	550	$\frac{1}{1600}$
3.	Mild steel	320	$\frac{1}{7500}$
4.	Timber	50	$\frac{1}{750}$

Q) The External and Internal diameter of a hollow cast iron column is 5cm and 4cm respectively. If the length of the column is 3m and both ends are fixed, Determine the crippling load using rankines formula. Take the value of σ_c as 550 N/mm^2 and 'α' as $\frac{1}{1600}$.

Given:- $l = 3m = 3000\,mm$

$D_o = 5cm = 50\,mm$

$D_i = 4cm = 40\,mm$

$\sigma_c = 550 \, N/mm^2$

$\alpha = \frac{1}{1600}$

w.k.t

$$p = \frac{\sigma_c A}{1 + \alpha\left(\frac{l_e}{k}\right)^2}$$

Get,

$$A = \frac{\pi}{4}\left[D_o^2 - D_i^2\right]$$

$$A = \frac{\pi}{4}\left[50^2 - 40^2\right]$$

$$A = 706.85 \, mm^2$$

Now,

$$I = \frac{\pi}{64}\left[D_o^4 - D_i^4\right]$$

$$= \frac{\pi}{64}\left[50^4 - 40^4\right]$$

$$I = 181132.45 \, mm^4$$

Now,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{181132.45}{706.85}}$$

$$k = 16 \, mm$$

As both ends of colum are fixed, $l_e = l/2$

$$l_e = \frac{3000}{2} = 1500.$$

Now, $$p = \frac{550 \times 706.85}{1 + \frac{1}{1600}\left(\frac{1500}{16}\right)^2}$$

$$p = 59873.35 \, N$$

$$p = 59.37 \, kN$$

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A 1.5m long column as a circular cross section of 5cm dia and one of the end of the column is fixed and other end is free. Taking factor of safety as 3, calculate the safe load by using Rankines formula. Take yield stress $\sigma_c = 560\ N/mm^2$ & $\alpha = \frac{1}{1600}$. Also calculate the crippling load & safe load by Euler's formula by taking young's modulus of cast iron as $1.2 \times 10^5\ N/mm^2$.

Sol^n:- Given

$l = 15m = 1500\ mm$, $\sigma_c = 560\ N/mm^2$

$D = 5cm = 50\ mm$ $\quad \alpha = \frac{1}{1600}$

$F.O.S = 3$

one end is fixed & other end is free

$l_e = 2l = 2 \times 1500 = 3000mm$

i) W.K.T

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{l_e}{k}\right)^2}$$

Now,

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 50^2 = 1963.49\ mm^2.$$

$$I = \frac{\pi}{64} D^4 = \frac{\pi}{64} \times 50^4 = 306796.15\ mm^4$$

Now,

$$k = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{306796.15}{1963.49}}$$

$$K = 12.50\ cm$$

Now, $$P = \frac{560 \times 1963.49}{1 + \frac{1}{1600}\left(\frac{3000}{12.5}\right)^2}$$

$\therefore \quad P = 29717.68 \text{ N}$

$= 29.71 \text{ KN}$

Now, safe load $= \dfrac{P}{F.O.S}$

$= \dfrac{29.71}{3}$

$= 9.90 \text{ KN}$

ii) wkt

$$P = \frac{\pi^2 EI}{l_e^2}$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 306796.15}{3000^2}$$

$P = 40372.75 \text{ N}$

$P = 40.37 \text{ KN}$

Now safe load $= \dfrac{P}{F.O.S}$

$= \dfrac{40.37}{3}$

$= 13.45 \text{ KN}$

* Straight line formula:- The Euler's formula and Rankine's formula give only the approximate values of crippling load due to following reasons.

i) The pin joints are not practically frictionless.

ii) The end fixation is never perfectly rigid.

iii) In case of Euler's formula, the effect of direct compression has been neglected.

iv) The load is not ~~at~~ exactly applied as desire

v) The members are never perfectly straight and uniform in Section

vi) The material of the members is not homogenous and isotropic.

On the ~~act~~ account of this,
the emperical straight line formula are commonly used in practical designing and it is given as

$$P = \sigma_c \times A - n\left(\frac{l_e}{K}\right) \times A.$$

$$P = A\left[\sigma_c - n\left(\frac{l_e}{K}\right)\right]$$

$$\frac{P}{A} = \sigma_c - n\left(\frac{l_e}{K}\right)$$

where P = crippling load on the column.

σ_c = compressive yielding stress.

A = Area of cross-section of the column.

$\frac{l_e}{K}$ = slenderness ratio.

n = A constant whose value depends upon material of the column.

* Johnson's parabolic formula:-

The critical load according to professor Johnson is given by $P = \sigma_c \times A - r\left(\frac{l_e}{K}\right) \times A.$

$$P = A\left[\sigma_c - r\left(\frac{l_e}{K}\right)\right]$$

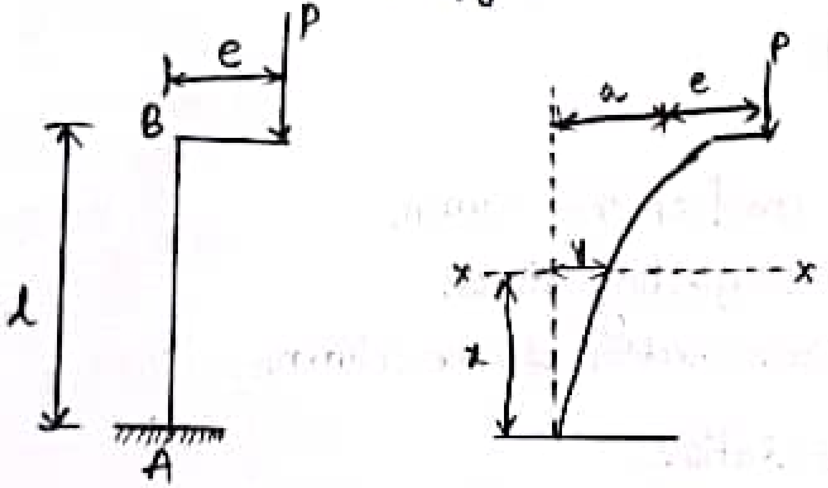
$$\frac{P}{A} = \sigma_c - \gamma\left(\frac{l_e}{k}\right)$$

where γ = a constant whose value depends upon material of the column

$$\gamma = \frac{\sigma_c}{4\pi^2 E^2}$$

* Columns with eccentric load :-

fig shows a column AB of length l which is fixed at A and free end at 'B'. the column is subjected to load P which is eccentric by an amount 'e'. The free end will sway sideways by an amount 'a' and the column will deflect as shown in the figure.



w.k.t

B.M @ x-x, $M = P((a+e)-y)$

$$M = P(a+e) - Py \quad \text{—} ①$$

But wkt

B.M, $M = EI\frac{d^2y}{dx^2}$ —②

①=②

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$$EI\frac{d^2y}{dx^2} = p(a+e) - py$$

$$EI\frac{d^2y}{dx^2} + py = p(a+e)$$

$$\frac{d^2y}{dx^2} + \frac{py}{EI} = \frac{p}{EI}(a+e)$$

The general ~~Section~~ solution for above DE is

$$y = c_1\cos\left(x\sqrt{\frac{p}{EI}}\right) + c_2\sin\left(x\sqrt{\frac{p}{EI}}\right) + (a+e) \longrightarrow (3)$$

At end A

At $x=0$, $y=0$

put above B.C in (3), we get

$$0 = c_1\cos\left(0\sqrt{\frac{p}{EI}}\right) + c_2\sin\left(0\sqrt{\frac{p}{EI}}\right) + (a+e)$$

$$0 = c_1 + (a+e)$$

$$c_1 = -(a+e)$$

Now w.r.to Diffenting (3) w.r.to x.

$$\frac{dy}{dx} = -c_1\sin\left(x\sqrt{\frac{p}{EI}}\right)\sqrt{\frac{p}{EI}} + c_2\cos\left(x\sqrt{\frac{p}{EI}}\right)\sqrt{\frac{p}{EI}} \rightarrow (4)$$

At end A.

$x=0$, $\frac{dy}{dx}=0$

$$0 = (a+e)\sin\left(0\sqrt{\frac{p}{EI}}\right)\sqrt{\frac{p}{EI}} + c_2\cos\left(0\sqrt{\frac{p}{EI}}\right)$$

$$0 = c_2$$

$$c_2 = 0$$

Now, At end B, $x=l$, $y=a$.

put above B.C in (3)

$$a = -(a+e)\cos\left(l\sqrt{\frac{p}{EI}}\right) + 0\sin\left(l\sqrt{\frac{p}{EI}}\right) + (a+e)$$

$$\cancel{a} = -(a+e)\cos\left(l\sqrt{\frac{p}{EI}}\right) + (\cancel{a}+e)$$

$$e = (a+e)\cos\left(l\sqrt{\frac{P}{EI}}\right)$$

$$(a+e) = \frac{e}{\cos\left(l\sqrt{\frac{P}{EI}}\right)}$$

$$(a+e) = e\sec\left(l\sqrt{\frac{P}{EI}}\right)$$

let us find the maximum compressive stress due to eccentricity there will be bending stress and also the direct stress.

$$\sigma_{max} = \sigma_d + \sigma_b$$

But

$$\sigma_d = \frac{P}{A}$$

w.k.t $$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = \frac{M}{Z}$$

Now, M = BM @ end A.

$$= P(a+e)$$

$$\sigma_b = \frac{P(a+e)}{Z}$$

Now $$\sigma_b = \frac{P \times e\sec\left(l\sqrt{\frac{P}{EI}}\right)}{Z}$$

Now, $$\sigma_{max} = \sigma_d + \sigma_b$$

$$\sigma_{max} = \frac{P}{A} + \frac{Pe\sec\left(l\cdot\sqrt{\frac{P}{EI}}\right)}{Z}$$

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The above eqn is used for a column whose one end is fixed and other end is free and load is eccentric to the column in this equation 'l' is the actual length of the column. The relation between the actual length and effective length of the column whose one end is fixed and other end is free. is $l_e = 2l$, $l = \frac{l_e}{2}$.

put above condition in above eqn.

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{Pe \sec\left(\frac{l_e}{2}\sqrt{\frac{P}{EI}}\right)}{Z}}$$

1. A circular column section is subjected to a load of 120 kN. The load is parallel to the axis but eccentric to an amount of 2.5mm. The external and internal diameter of the column is 60mm and 50mm. If both ends of the column ~~is 60mm and 5~~ are hinged and the column is 2.1 m long. then Determine the max. stress in the column. Take $E = 2\times10^5$ N/mm

Soln: Given $P = 120kN = 120\times10^3$ N, $e = 2.5mm$, $D_o = 60mm$, $D_i = 50mm$

$l = 2.1m = 2100mm$,

w.k.t, $\sigma_{max} = \frac{P}{A} + \frac{Pe \sec\left(\frac{l_e}{2}\sqrt{P/EI}\right)}{Z}$

$A = \pi/4 (D_o^2 - D_i^2)$

$A = \pi/4 (60^2 - 50^2)$

$A = 863.93 \ mm^2$

Now, $I = \pi/64 (D_o^4 - D_i^4) = \pi/64 (60^4 - 50^4)$

$= 32937635 \ mm^4$

Now, $Z = I/y_{max} = 329376.35 / D_o/2$

$= 10979.81\ mm^3$.

Hence, both the ends of the column are hinged

So, $l_e = l$.

$$\sigma_{max} = \frac{P}{A} + \frac{Pe \sec\left(\frac{l_e}{2}\sqrt{P/EI}\right)}{Z}$$

$$\sigma_{max} = \frac{120\times10^3}{863.93} + \frac{120\times10^3 \times 2.5 \sec\left(\frac{2100}{2}\sqrt{\frac{120\times10^3}{2\times10^5\times329376.35}}\right)}{10979.81}$$

$$\sigma_{max} = 138.9 + 27.32 \times \sec(1.417)$$

$$\sigma_{max} = 138.9 + 27.32 \times \sec(1.417 \times 180/\pi)$$

$$\sigma_{max} = 138.9 + 27.32 \times \frac{1}{\cos(81.36)}$$

$$\sigma_{max} = 317.07\ N/mm^2.$$

But $k = \sqrt{I/m} \rightarrow k^2 = I/m \rightarrow I = k^2 m$

Now, $Z = Ak^2/y_e$. $\longrightarrow$ prof. perry's formula

Substitute Z in above eqn.

$$\sigma_{max} = \frac{P}{A} + \frac{Pe \sec\left(\frac{l_e}{2}\sqrt{P/EI}\right)}{Ak^2/y_e}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Pe\, y_e \sec\left(\frac{l_e}{2}\sqrt{P/EI}\right)}{Ak^2} \longrightarrow (1)$$

Now, $\sigma_d = P/A$.

$$P_e = \frac{\pi^2 EI}{l_e^2}$$

$$l_e^2 = \frac{\pi^2 EI}{P_e}$$

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$$l_e = \sqrt{\frac{\pi^2 EI}{P_e}}$$

$$l_e = \pi\sqrt{EI/P_e} \quad \text{sub in eqn (1)}$$

$$\sigma_{max} = P/A + \frac{P e y_c \sec\left(\pi/2 \sqrt{\frac{EI}{P_e}} \times \sqrt{P/EI}\right)}{Ak^2}$$

$$= P/A\left(1 + \frac{e y_c \sec\left(\pi/2 \sqrt{P/EI}\right)}{k^2}\right)$$

$$\sigma_{max} = \sigma_d\left(\frac{1 + e y_c \sec\left(\pi/2\sqrt{P/EI}\right)}{k^2}\right) \longrightarrow (2)$$

But according to prof. perry's

$$\sec\left(\pi/2\sqrt{P/P_e}\right) = 1.2\, P_e/P_e - P$$

$$= \frac{1.2\sigma_e A}{\sigma_e A - \sigma_A}$$

$$\sec\left(\pi/2\sqrt{P/P_e}\right) = \frac{1.2\sigma_e}{\sigma_e - \sigma} \longrightarrow (3)$$

Now, put (3) in (2) we get

$$\sigma_{max} = \sigma_d\left(1 + \frac{e y_e}{k^2} \cdot \frac{1.2\sigma_e}{\sigma_e - \sigma}\right)$$

$$\sigma_{max}/\sigma_d = 1 + \frac{e y_e}{k^2} \times \frac{1.2\sigma_e}{\sigma_e - \sigma})$$

$$\left(\frac{\sigma_{max}}{\sigma_d} - 1\right)\left(\frac{\sigma_e - \sigma}{\sigma_e}\right) = \frac{1.2\sigma_e}{\sigma_e - \sigma}$$

$$\left(\frac{\sigma_{max} - 1}{\sigma_d}\right)\left(\frac{\sigma_c - \sigma}{\sigma_e}\right) = \frac{1.2 e y_e}{k^2}$$

* Prof. Perry's formula :-

In cases where we have to determine the safe load that can be applied on a column at a given eccentricity then Prof. Perry's formula proves to be quiet useful.

σ_d = direct stress

σ_{max} = max. permissible stress.

σ_b = max. compressive stress due to bending.

y_c = distance from neutral axis to the extreme layer in compression

W.K.T, $\sigma_{max} = \sigma_d + \sigma_b$

$$\sigma_{max} = \frac{P}{A} + \frac{P \cdot e \sec\left(\frac{l_e}{2}\sqrt{P/EI}\right)}{Z}$$

But $Z = I/y_c$.

* Beam columns :-

Columns carry axial compressive loads. If the columns are also subjected to transverse loads, then they are known as beam columns. The transverse load is generally uniformly distributed.

But, let us consider 2 cases when i) transverse lad ladd load is a point load and acts at a centre.
ii) transverse load is uniformly distributed load.

* Strut subjected to compressive axial load / axial thrust and transverse point load at the centre :-

Figure shows a strut AB of length 'l' subjected to axial compressive load P and a transverse point load W at a centre. The strut is pinned at both the ends. consider, a section x-x at a distance 'x' from end A. let y is the deflection at this section. BM at this section is given by :-

Now, BM @ x-x, $M = -R_A x - Py$

$$M = \frac{-W}{2}x - Py \longrightarrow (1)$$

But WKT, BM, $M = EI\frac{d^2y}{dx^2} \longrightarrow (2)$

(1) = (2)

$$EI\frac{d^2y}{dx^2} = -\frac{Wx}{2} - Py$$

$$EI\frac{d^2y}{dx^2} + Py = -\frac{Wx}{2}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = -\frac{Wx}{2EI}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{-Wx}{2P} \times \frac{P}{EI}$$

The solution for above DE is

$$y = C_1\cos\left(x\sqrt{P/EI}\right) + C_2\sin\left(x\sqrt{P/EI}\right) - \frac{Wx}{2P} \longrightarrow (3)$$

Diff. (3) w.r.to 'x'

$$\frac{dy}{dx} = -C_1\sin\left(x\sqrt{P/EI}\right)\sqrt{P/EI} + C_2\cos\left(x\sqrt{P/EI}\right)\left(\sqrt{P/EI}\right) - \frac{W}{2P} \longrightarrow (4)$$

At x=0, y=0, put above BC in (3)

$$0 = C_1\cos\left(0\sqrt{P/EI}\right) + C_2\sin\left(0\sqrt{P/EI}\right) - \frac{W \times 0}{2P}$$

$$C_1 = 0$$

Now, At $x = l/2$, $\frac{dy}{dx} = 0$

put above BC in (4)

$$0 = 0\sin\left(\tfrac{l}{2}\sqrt{P/EI}\right)\sqrt{P/EI} + C_2\cos\left(\tfrac{l}{2}\sqrt{P/EI}\right)\sqrt{\frac{P}{EI}} - \frac{W}{2P}.$$

$$0 = C_2 \cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} - \frac{\omega}{2P}$$

$$C_2 = \frac{\omega}{2P}\sqrt{\frac{EI}{P}}\ \sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)$$

Now, put c_1 and c_2 in eqn ③

$$y = 0\cos\left(0\sqrt{\frac{P}{EI}}\right) + \frac{\omega}{2P}\sqrt{\frac{EI}{P}}\ \sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sin\left(x\sqrt{\frac{P}{EI}}\right) - \frac{\omega x}{2P}$$

$$y = \frac{\omega}{2P}\sqrt{\frac{EI}{P}}\ \sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sin\left(x\sqrt{\frac{P}{EI}}\right) - \frac{\omega x}{2P} \longrightarrow ⑤$$

y Eqn ⑤ is general equation for deflection

Now, for max. deflection put $x = \frac{l}{2}$ in eqn ⑤

$$y_{max} = \frac{\omega}{2P}\sqrt{\frac{EI}{P}}\ \sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) \times \sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - \frac{\omega}{2P} \times \frac{l}{2}$$

$$\boxed{y_{max} = \frac{\omega}{2P}\sqrt{\frac{EI}{P}}\tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - \frac{\omega l}{4P}}$$

* **Max. Bending Moment :-**

from ①, $M = -\left(Py + \frac{\omega}{2}x\right)$

In order to get max. bending moment

put $x = \frac{l}{2}$, $y = y_{max}$ in above eqn

$$M_{max} = -\left(Py_{max} + \frac{\omega}{2} \times \frac{l}{2}\right)$$

$$= -\left(P \times \left(\frac{\omega}{2P}\sqrt{\frac{EI}{P}}\tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - \frac{\omega l}{4P}\right) + \frac{\omega l}{4}\right)$$

$$= -\left(\frac{\omega}{2}\sqrt{\frac{EI}{P}}\ \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\right)$$

-ve sign is due to sign convention, Hence magnitude of max. BM is given as,

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$$\boxed{M_{max} = \frac{w}{2}\sqrt{\frac{EI}{P}} \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}$$

Max. stress :- $\sigma_{max} = \sigma_d + \sigma_b$

but $\sigma_d = P/A$

Now WKT, $M/I = \sigma_b/y$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = My_c/Ak^2$$

for max. stress put $M = M_{max}$, $\sigma_b = M_{max}\, y_c/Ak^2$

$$\sigma_b = \frac{w}{2}\sqrt{\frac{EI}{P}} \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) \times y_c/Ak^2$$

$$\sigma_{max} = \sigma_b + \sigma_d$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \left[\frac{w}{2}\sqrt{\frac{EI}{P}} \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\right] \times y_c/Ak^2}$$

Q) Determine the maximum stress induced in a cylindrical steel strut of length 1.2 m and diameter 30 mm. The strut is hinged at both of its ends and subjected to an axial thrust of 20 kN at its ends and a transverse point load of 1.8 kN at the centre. Take E = 208 GN/m²

Given :-

$l = 1.2\,m = 1200\,mm$

$d = 30\,mm$

$P = 20\,kN = 20 \times 10^3\,N$

$W = 1.8\,kN = 1.8 \times 10^3\,N$

$E = 208\,GN/m^2$

$= 208 \times 10^9\,N/m^2 \Rightarrow 208 \times 10^9 \times 10^{-6}\,N/mm^2$

$E = 208 \times 10^3\,N/mm^2 = 2.08 \times 10^5\,N/mm^2$

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Q.No.4

$$\sigma_{max} = \frac{P}{A} + \left[\frac{w}{2}\sqrt{\frac{EI}{P}}\tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\right] \times \frac{y_c}{Ak^2}$$

Now, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times 30^2 = 706.25\ mm^2$

$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}(30)^4 = 39760.78\ mm^4$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{39760.78}{706.25}} = 7.5\ mm$

Now

$$\sigma_{max} = \frac{20\times10^3}{706.25} + \left[\frac{1.8\times10^3}{2}\times\sqrt{\frac{2.08\times10^5\times 39760.78}{20\times10^3}}\tan\left(\frac{1200}{2}\sqrt{\frac{20\times10^3}{2.08\times10^5\times39760.78}}\right)\right]\times\frac{15}{706.25\times7.5^2}$$

$$\sigma_{max} = 28.29 + \left[578714.16\tan\left(0.933\times\frac{180}{\pi}\right)\right]\times 3.77\times10^{-4}$$

$\sigma_{max} = 28.29 + 294.51$

$\sigma_{max} = 322.8\ N/mm^2$ //

* Strut subjected to compressive axial load or axial thrust and a transverse uniformly Distributed load of intensity w per unit length.

⇒ figure shows a strut AB of length 'l' subjected to axial thrust 'P'. at its ends. and also a transverse UDL of intensity w per unit length.

The strut is pinned at both its ends.

Consider a section xx at a distance x from end A and

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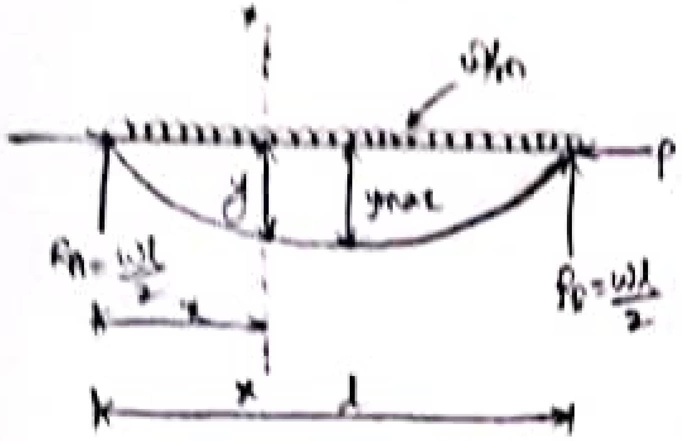
let y be the deflection at this section and the B.M at this section is given as.



B.M @ x-x, $M = -R_A x - Py + wx \times \frac{x}{2}$

$$M = -\frac{wl}{2}x - Py + \frac{wx^2}{2}$$

$$M = -Py - \frac{wl}{2}x + \frac{wx^2}{2} \longrightarrow (1)$$

Now, wkt.

B.M, $M = EI\frac{d^2y}{dx^2} \longrightarrow (2)$

Now, Diff (1) w.r.to x.

$$\frac{dM}{dx} = -P\frac{dy}{dx} - \frac{wl}{2} + \frac{wx\,2x}{2}$$

$$\frac{dM}{dx} = -P\frac{dy}{dx} + wx - \frac{wl}{2} \longrightarrow (3)$$

Diff (3) w.r.to x.

$$\frac{d^2M}{dx^2} = -P\frac{d^2y}{dx^2} + w \longrightarrow (4)$$

Now, from (2)

$$M = EI\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

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put above condition in ④

$$\frac{d^2M}{dx^2} = -P\frac{M}{EI} + \omega.$$

$$\frac{d^2M}{dx^2} + \frac{P}{EI}.M = \frac{\omega P}{EI} \times \frac{EI}{P}$$

$$\frac{d^2M}{dx^2} + \frac{P}{EI}.M = \frac{\omega EI}{P} \times \frac{P}{EI}$$

Now. sol. for above D.E is

$$M = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{\omega EI}{P} \longrightarrow ⑤$$

Diff. ⑤ w.r.to x.

$$\frac{dM}{dx} = -C_1 \sin\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} \longrightarrow ⑥$$

Now, At $x=0$, $M=0$

put above condition in ⑤

$$0 = C_1 \cos\left(0\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(0\sqrt{\frac{P}{EI}}\right) + \frac{\omega EI}{P}$$

$$0 = C_1 + \frac{\omega EI}{P}$$

$$C_1 = -\frac{\omega EI}{P}$$

Now, At $x = \frac{l}{2}$, $\frac{dM}{dx} = 0$

$$0 = \frac{\omega EI}{P} \sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + C_2 \cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}}$$

$$0 = \frac{\omega EI}{P} \sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) + C_2 \cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)$$

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$$c_2 \cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) = -\frac{WEI}{P}\sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)$$

$$C_2 = \frac{-WEI}{P}\tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)$$

Now put c_1 & c_2 in (5), we get.

$$M = -\frac{WEI}{P}\cos\left(x\sqrt{\frac{P}{EI}}\right) - \frac{WEI}{P}\tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{WEI}{P}$$

$$M = -\frac{WEI}{P}\left[\cos\left(x\sqrt{\frac{P}{EI}}\right) + \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sin\left(x\sqrt{\frac{P}{EI}}\right) - 1\right] \longrightarrow (7)$$

Max. B.M:-

In order to get max. B.M put $x = \frac{l}{2}$ in (7).

$$M_{max} = -\frac{WEI}{P}\left[\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) + \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]$$

$$M_{max} = -\frac{WEI}{P}\left[\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) + \frac{\sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}{\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}\sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]$$

$$M_{max} = -\frac{WEI}{P}\left[\frac{\cos^2\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) + \sin^2\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}{\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)} - 1\right]$$

$$M_{max} = -\frac{WEI}{P}\left[\frac{1}{\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)} - 1\right]$$

$$\boxed{M_{max} = -\frac{WEI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]}$$

Max. deflection:-

In order to get max. deflection

put $x = l/2$ & $M = M_{max}$ in ①

$$M = -Py + \frac{wx^2}{2} - \frac{wl}{2}x$$

$$M_{max} = -Py_{max} + \frac{wx^2}{2} - \frac{wl}{2}x$$

$$-\frac{wEI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] = -P \times y_{max} + \frac{w}{2} \times \left(\frac{l}{2}\right)^2 - \frac{wl}{2}\left(\frac{l}{2}\right)$$

$$= -Py_{max} + \frac{wl^2}{8} - \frac{wl^2}{4}$$

$$-\frac{wEI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] = -Py_{max} - \frac{wl^2}{8}$$

$$Py_{max} = \frac{wEI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] - \frac{wl^2}{8}$$

$$\boxed{y_{max} = \frac{wEI}{P^2}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] - \frac{wl^2}{8}}$$

Max. stress:-

$$\sigma_{max} = \sigma_d + \sigma_b$$

But $\sigma_d = \frac{P}{A}$

$$\sigma_b = \frac{My_c}{I}$$

$$\sigma_b = M_{max} \times \frac{y_c}{I}$$

$$\sigma_b = \frac{wEI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] \times \frac{y_c}{I}$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{wEI}{P}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] \times \frac{y_c}{I}}$$

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1) Determine the maximum stress induced in a horizontal strut of length 2.5 m and of rectangular cross section 40 mm wide & 80 mm deep when it carries an axial thrust of 100kN and vertical load of 6 kN/m length. The strut is having pinned joints at its ends. Take $E = 208\ GN/m^2$.

Soln:- Given = $l = 2.5m = 2500\,mm$

$b = 40\,mm$

$d = 80\,mm$

$P = 100kN = 100 \times 10^3\ N$

$\omega = 6\ kN/m = 6 \times 10^3 / 10^3 = 6\ N.$

$E = 208 \times 10^3\ N/mm^2 = 2.08 \times 10^5\ N/mm^2.$

$I = \frac{bd^3}{12} = \frac{40 \times (80)^3}{12} = 1706666.667\ mm^4$

$A = b \times d = 3200.\ mm^2$

Now

$$\sigma_{max} = \frac{P}{A} + \frac{\omega EI}{P^2}\left[\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] \times \frac{\omega l^2}{8} \times \frac{y_c}{I}.$$

$$\sigma_{max} = \frac{100 \times 10^3}{3200} + \frac{6 \times 2.08 \times 10^5 \times 1706666.667}{(100 \times 10^3)^2}\left[\sec\left[\frac{2500}{2}\sqrt{\frac{100 \times 10^3}{2.08 \times 10^5 \times 1706666.667}} \times \frac{180}{\pi} - 1\right] \times \frac{6 \times (2500)^2}{8} \cdot \frac{40}{1706666.667}\right.$$

$\sigma_{max} = 31.25 + 21299199.92 \times \sec\left(0.663 \times \frac{180}{\pi}\right) - 1) \times 2.34 \times 10^{-5}$

$= 31.25 + 21299199.92 \times (\sec(37.98) - 1) \times 2.34 \times 10^{-5}$

$= 31.25 + 21299199.92 \times [1.2687 - 1] \times 2.34 \times 10^{-5}$

0.2687

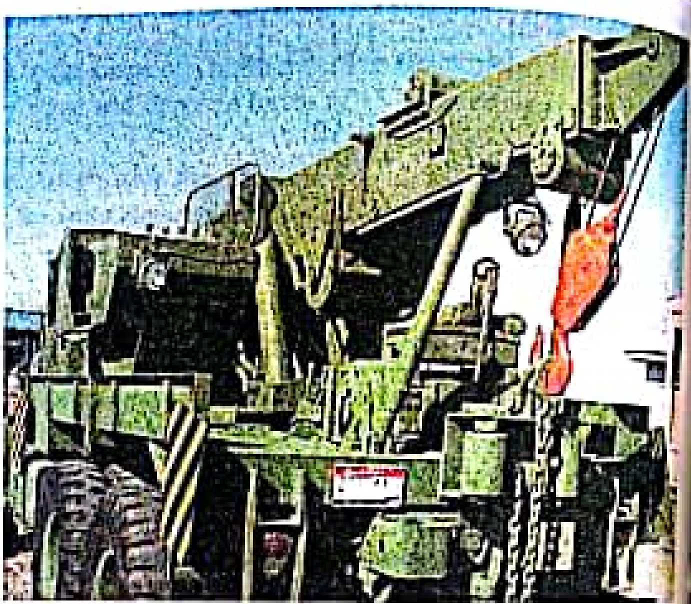
$\boxed{\sigma_{max} = 165.170\ N/mm^2}.$

MODULE-V

UNSYMMETRICAL BENDING AND SHEAR CENTRE

Chapter 21

Unsymmetrical Bending and Shear Centre



21·1. INTRODUCTION

In chapter 5 (Bending stresses), while using the well known bending equation $\frac{M}{I} = \frac{\sigma}{y}$, it is assumed that the neutral axis of the cross-section of the beam is perpendicular to the plane of loading. This condition implies that the plane of loading or plane of bending, is coincident with, or parallel to, a plane containing a principal centroidal axis of inertia of the cross-section of the beam. *If, however, the plane of loading or that of bending, does not lie in (or parallel to) a plane that contains the principal centroidal axis of the cross-section, the bending is called* **unsymmetrical bending.**

In the case of unsymmetrical bending, the direction of neutral axis is *not* perpendicular to the plane of bending.

Following are the two *reasons* of unsymmetrical bending :

(*i*) The section is symmetrical (*viz.* rectangular, circular, I sections) but the load line is inclined to both the principal axes.

(*ii*) The section itself is unsymmetrical (*viz.* angle section or channel section vertical web) and the load line is along any centroidal axis.

21·2. PRODUCT OF INERTIA

21·2·1. Parallel Axes Theorem for Product of Inertia

21·2·2. Principal Axes and Principal Moments of Inertia

Refer Articles 3·16·1 and 3·16·2 (chapter 3) and Examples 3·17, 3·18 and 3·19.

21·3. STRESSES DUE TO UNSYMMETRICAL BENDING

Fig. 21·1 shows the cross-section of a beam under the action of a bending moment M acting in plane YY.

Also, G = Centroid of the section,

XX, YY = Co-ordinate axes passing through G, and

UU, VV = Principal axes inclined at an angle θ to XX and YY axes respectively.

Let us determine the stress distribution over the section.

The moment M in the plane YY can be resolved into its components in the planes UU and VV as follows :

Moment in the plane UU, $M' = M \sin \theta$ ...(21·1)

Moment in the plane VV, $M'' = M \cos \theta$ ...(21·2)

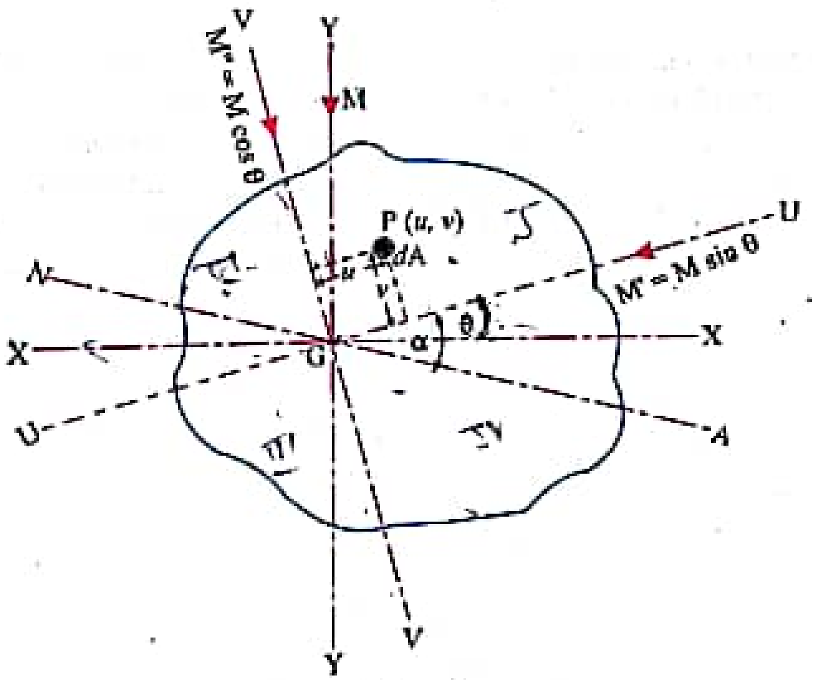


Fig. 21.1

The components M' and M'' have their axes along VV and UU respectively.

The resultant bending stress at the point $P(u, v)$ is given by,

$$\sigma_b = \frac{M' \cdot u}{I_{VV}} + \frac{M'' \cdot v}{I_{UU}} = \frac{M \sin\theta . u}{I_{VV}} + \frac{M \cos\theta . v}{I_{UU}}$$

or,

$$\sigma_b = M\left[\frac{v \cos\theta}{I_{UU}} + \frac{u \sin\theta}{I_{VV}}\right] \qquad ...(21·3)$$

At any point the nature of σ_b will depend upon the quadrant in which it lies. In other words the signs of u and v will have to be taken into account while determining the resultant bending stress.

The equation of the neutral axis (N.A.) can be found by finding the *locus of the points on which the resultant stress is zero*. Thus the points lying on neutral axis will satisfy the condition that $\sigma_b = 0$,

i.e. $$M\left[\frac{v\cos\theta}{I_{UU}} + \frac{u\sin\theta}{I_{VV}}\right] = 0 \quad \text{...(From eqn. 21.3)}$$

or, $$\frac{v\cdot\cos\theta}{I_{UU}} + \frac{u\sin\theta}{I_{VV}} = 0$$

or, $$v = -\left[\frac{I_{UU}}{I_{VV}} \times \frac{\sin\theta}{\cos\theta}\right]u$$

or, $$v = -\left[\frac{I_{UU}}{I_{VV}}\tan\theta\right]u \quad \text{...(21.4)}$$

This is an equation of a straight line passing through the centroid G of the section and inclined at an angle α with UU where,

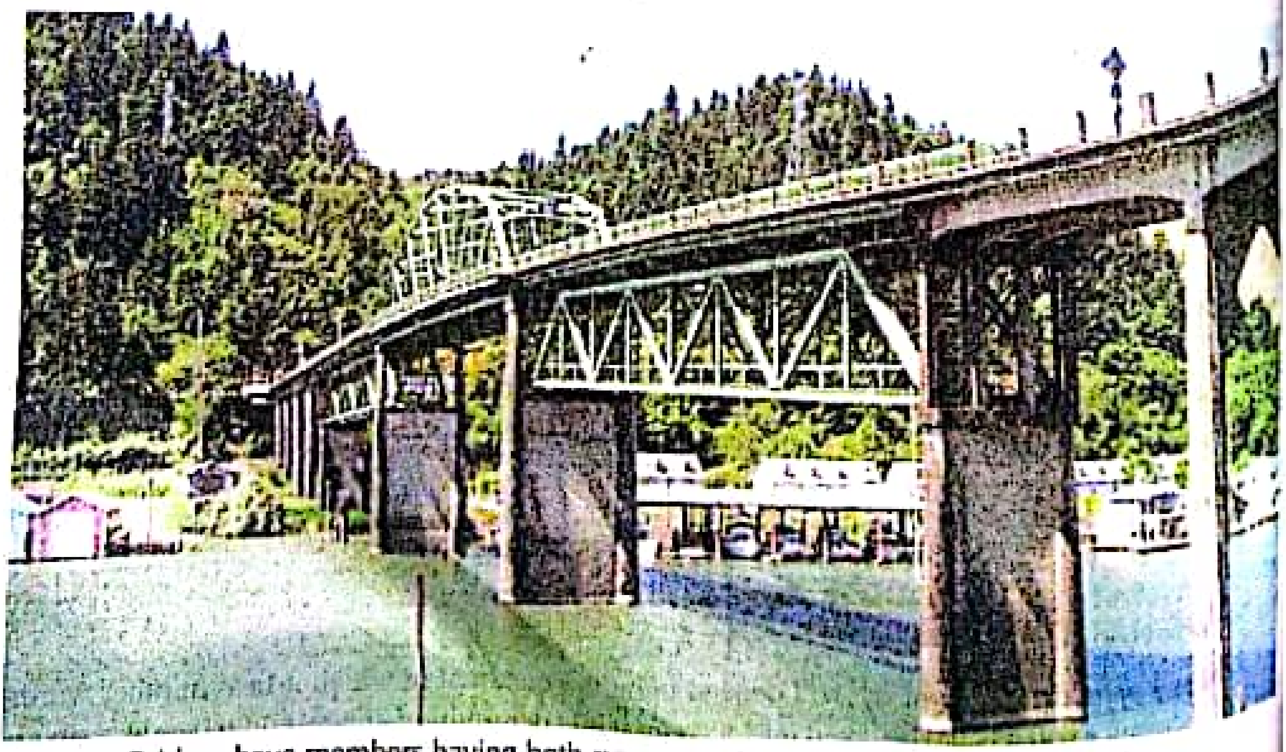
$$\tan\alpha = -\left[\frac{I_{UU}}{I_{VV}}\tan\theta\right] \quad \text{...(21.5)}$$

Following points are worth noting :

(*i*) The maximum stress will occur at a point which is at the greatest distance from the neutral axis.

(*ii*) All the points of the section on one side of the neutral axis will carry stresses of the same nature and on the other side of its axis, of opposite nature.

(*iii*) In the case where there is direct stress in addition to the bending stress, the neutral axis will still be a straight line but will *not* pass through G (centroid of the section). This is obvious from the fact that for finding the equation of the neutral axis the resultant stress which is the algebraic sum of direct and bending stresses will be equated to zero. This has already been discussed in chapter 6, the only difference being that I_{XX} and I_{YY} be replaced by I_{UU} and I_{VV} respectively and x and y to be replaced by u and v respectively.



Bridges have members having both symmetrical and unsymmetrical cross-sections.

21·4. DEFLECTION OF BEAMS DUE TO UNSYMMETRICAL BENDING

Fig. 21·2. shows the transverse section of the beam with centroid G. XX and YY are two rectangular co-ordinate axes and UU and VV are the principal axes inclined at an angle θ to the XY set of co-ordinate axes. W is the load acting along line YY on the section of the beam. The load W can be resolved into the following two components :

(i) $W \sin\theta$.... along UG

(ii) $W \cos\theta$.... along VG.

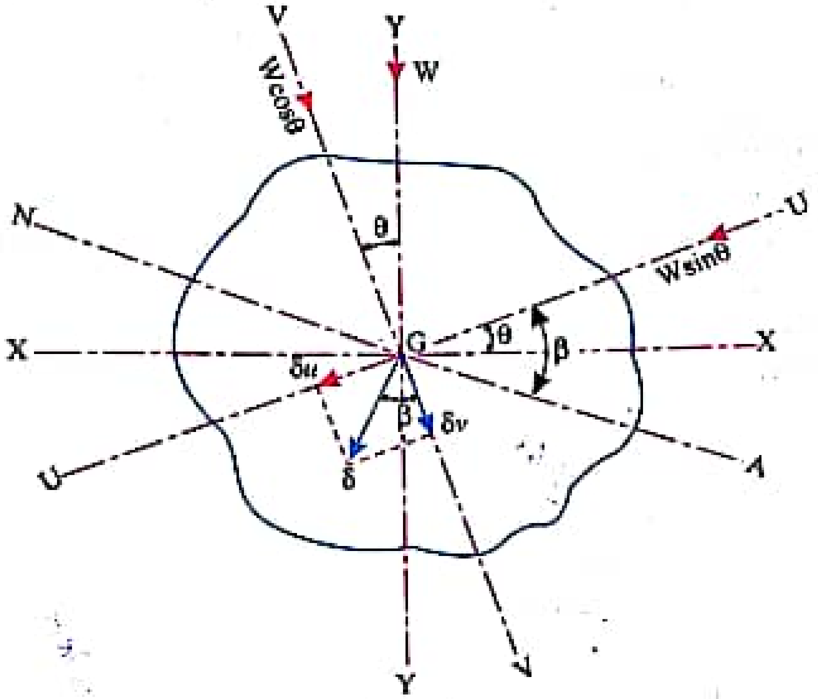


Fig. 21·2

Let, δu = Deflection caused by the component $W \sin\theta$ along the line GU for its bending about VV axis, and

δv = Deflection caused by the component $W \cos\theta$ along the line GV due to bending about UU axis.

Then, depending upon the *end conditions* of the beam, the values of δu and δv are given by:

$$\delta u = \frac{K\,(W \sin\theta)\, l^3}{EI_{VV}} \qquad ...(21\cdot5)$$

and,

$$\delta v = \frac{K\,(W \cos\theta)\, l^3}{EI_{UU}} \qquad ...(21\cdot6)$$

where, K = A constant depending on the end conditions of the beam and position of the load along the beam, and

l = Length of the beam.

The total or resultant deflection δ can then be found as follows :

$$\delta = \sqrt{(\delta u)^2 + (\delta v)^2}$$

$$= \frac{Kl^3}{E}\sqrt{\left(\frac{(W \sin\theta)}{I_{VV}}\right)^2 + \left(\frac{(W \cos\theta)}{I_{UU}}\right)^2}$$

or, $\delta = \frac{KWl^3}{E}\sqrt{\frac{\sin^2\theta}{I_{VV}^2} + \frac{\cos^2\theta}{I_{UU}^2}}$ ...(21·7)

The inclination β of the deflection δ, with the line GV is given by :

$$\tan\beta = \frac{\delta u}{\delta v} = \frac{I_{UU}}{I_{VV}}\tan\theta \quad ...(21·8)$$

From eqns. (21·5) and (21·8) it is evident that the magnitudes of α and β are the same and are measured from perpendicular lines (GU and GV) in same direction as shown in Figs. 21·1 and 21·2. Thus the deflection δ will be in a direction *perpendicular to the neutral axis.*

Example 21·1. *A 80 mm × 80 mm × 10 mm angle section shown in Fig. 21·3 is used as a simply supported beam over a span of 2·4 m. It carries a load of 400 N along the line YG, where G is the centroid of the section. Calculate :*

(i) *Stresses at the points A, B and C of the mid section of the beam;*

(ii) *Deflection of the beam at the mid section and its direction with the load line;*

(iii) *Position of the neutral axis.*

Take : E = 200 GN/m².

Solution. Refer Fig. 21·3

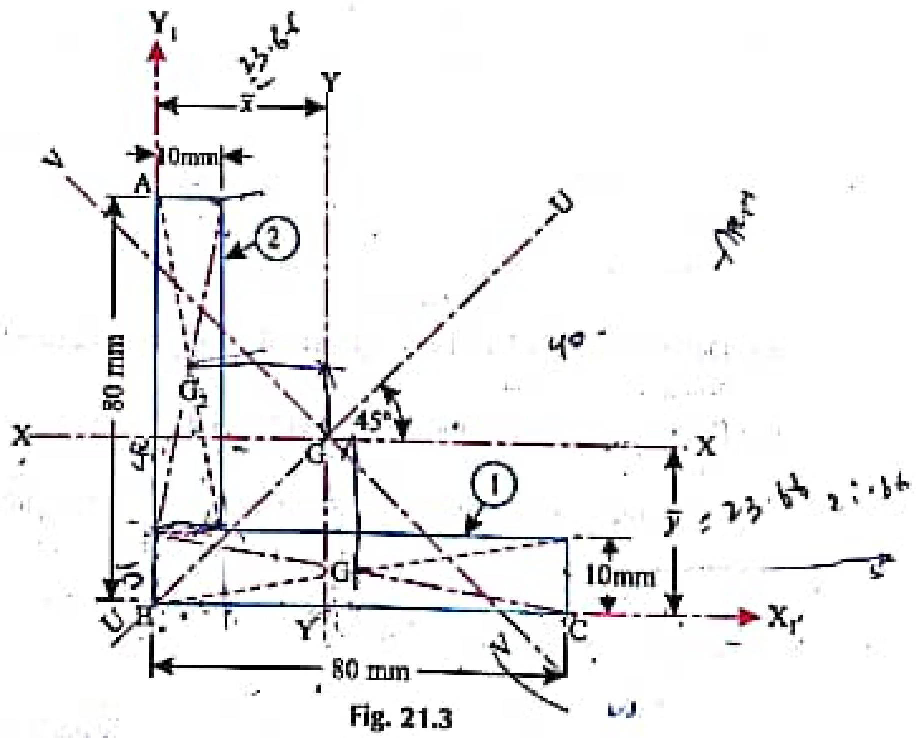


Fig. 21.3

Let $(\bar{x}, \bar{y})$ be the co-ordinates of centroid G, with respect to the rectangular axes BX_1 and BY_1.

Now $\bar{x}\ (= \bar{y}) = \frac{80\times10\times40 + 70\times10\times5}{80\times10 + 70\times10} = \frac{32000 + 3500}{800 + 700} = 23·66$ mm

Moment of inertia about XX axis.

$$I_{XX} = \left[\frac{80\times10^3}{12} + 80\times10\times(23·66-5)^2\right] + \left[\frac{10\times70^3}{12} + 70\times10\times(45-23·66)^2\right]$$

$$= [6666·66 + 278556] + [285833·33 + 318777] = 889833 \text{ mm}^4$$

$$= 8·898\times10^5 \text{ mm}^4 = I_{YY} \text{ (since it is an equal angle section)}$$

Co-ordinates of G_1 = $+(40-23.66), -(23.66-5) = (16.34, -18.66)$

Co-ordinates of G_2 = $-(23.66-5), +(45-23.66) = (-18.66, +21.34)$

Product of inertia, $I_{XY} = 80 \times 10\,(16.34)(-18.66) + 70 \times 10\,(-18.66)(21.34)$

$$= -243923.5 - 278743 = -522666 \text{ mm}^4$$

$$= -5.2266 \times 10^5 \text{ mm}^4$$

[Product of inertia about the centroid axes is zero because portions 1 and 2 are rectangular strips.]

If θ is the inclination of principal axes with GX, passing through G then,

$$\tan 2\theta = \frac{2I_{XY}}{I_{YY} - I_{XX}} = \infty = \tan 90^\circ. \qquad (\because I_{XX} = I_{YY})$$

$$\therefore \quad 2\theta = 90^\circ$$

i.e. $\theta_1 = 45^\circ$ and $\theta_2 = 90^\circ + 45^\circ = 135^\circ$ are the inclinations of the principal axes GU and GV respectively.

Principal moment of inertia :

$$I_{UU} = \frac{1}{2}(I_{XX} + I_{YY}) + \frac{1}{2}(I_{XX} - I_{YY})\cos 90^\circ - I_{XY}\sin 90^\circ \quad (\text{At } \theta_1 = 45^\circ)$$

$$= \frac{1}{2}(8.898 + 8.898) \times 10^5 + \frac{1}{2} \times 0 \times \cos 90^\circ - (-5.2266 \times 10^5)$$

$$= (8.898 + 5.2266) \times 10^5 = 14.1246 \times 10^5 \text{ mm}^4$$

Also, $I_{UU} + I_{VV} = I_{XX} + I_{YY}$

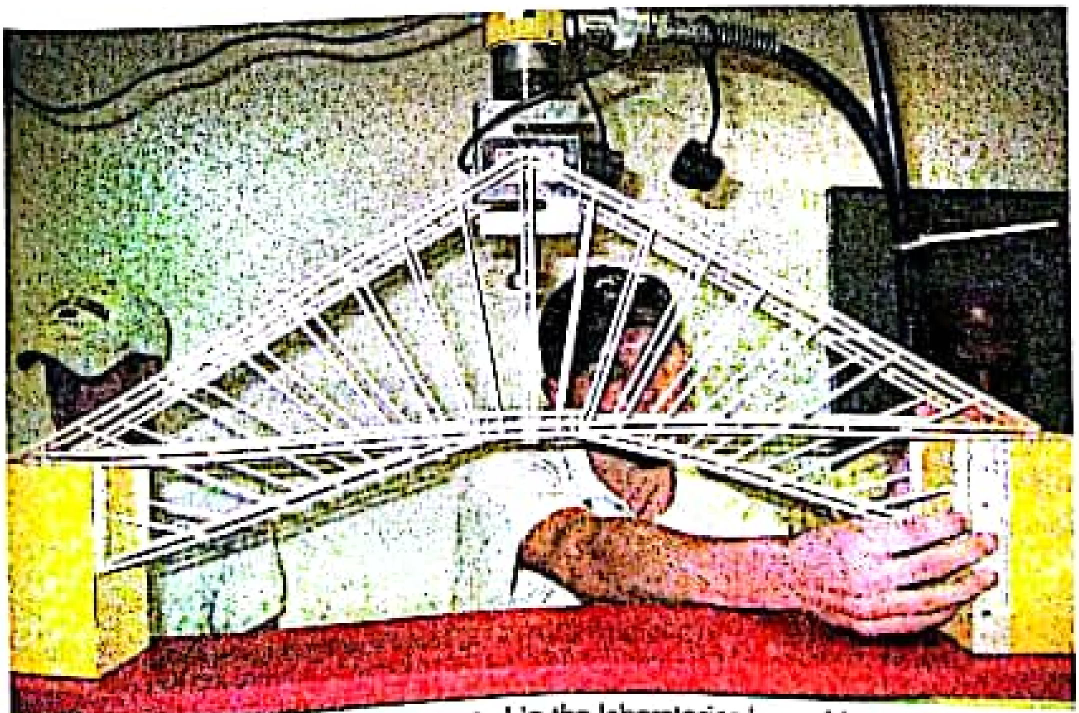
or, $I_{VV} = I_{XX} + I_{YY} - I_{UU}$

$$= 2 \times 8.898 \times 10^5 - 14.1246 \times 10^5 = 3.67 \times 10^5 \text{ mm}^4$$

(i) Stresses at the points *A*, *B* and *C* :

Bending moment at the mid-section,

$$M = \frac{Wl}{4} = \frac{400 \times 2.4 \times 10^3}{4} = 2.4 \times 10^5 \text{ Nmm}$$



Complex frameworks are often tested in the laboratories by making prototypes, before they are actually implemented.

The components of the bending moments are :

$$M' = M\sin\theta = 2{\cdot}4\times10^5 \sin 45° = 1{\cdot}697\times10^5 \text{ Nmm}$$

$$M'' = M\cos\theta = 2{\cdot}4\times10^5 \cos 45° = 1{\cdot}697\times10^5 \text{ Nmm}$$

u, v co-ordinates :

Point A. $x = -23{\cdot}66,\ y = 80 - 23{\cdot}66 = 56{\cdot}34$ mm

$$u = x\cos\theta + y\sin\theta$$
$$= -23{\cdot}66\times\cos 45° + 56{\cdot}34\times\sin 45° = 23{\cdot}1 \text{ mm}$$
$$v = y\cos\theta - x\sin\theta$$
$$= 56{\cdot}34\cos 45° - (-23{\cdot}66\times\sin 45°) = 56{\cdot}56 \text{ mm}$$

Point B. $x = -23{\cdot}66,\ y = -23{\cdot}66$ mm

$$u = -23{\cdot}66\times\cos 45° + (-23{\cdot}66\sin 45°) = -33{\cdot}45 \text{ mm}$$
$$v = -23{\cdot}66\cos 45° - (-23{\cdot}66\sin 45°) = 0$$

Point C. $x = 80 - 23{\cdot}66 = 56{\cdot}34,\ y = -23{\cdot}66$ mm

$$u = 56{\cdot}34\times\cos 45° - 23{\cdot}66\sin 45° = 23{\cdot}1 \text{ mm}$$
$$v = -23{\cdot}66\cos 45° - 56{\cdot}34\sin 45° = -56{\cdot}56 \text{ mm}$$

$$\sigma_A = \frac{M'u}{I_{VV}} + \frac{M''v}{I_{UU}}$$

$$= 1{\cdot}697\times10^5\left[\frac{23{\cdot}1}{3{\cdot}67\times10^5} + \frac{56{\cdot}56}{14{\cdot}1246\times10^5}\right] = 17{\cdot}47 \text{ N/mm}^2$$

i.e. $\sigma_A = 17{\cdot}47$ N/mm² **(Ans.)**

$$\sigma_B = 1{\cdot}697\times10^5\left[\frac{-33{\cdot}45}{3{\cdot}67\times10^5} + \frac{0}{14{\cdot}1246\times10^5}\right] = -15{\cdot}47 \text{ N/mm}^2$$

i.e. $\sigma_B = -15{\cdot}47$ N/mm² **(Ans.)**

$$\sigma_C = 1{\cdot}697\times10^5\left[\frac{23{\cdot}1}{3{\cdot}67\times10^5} - \frac{56{\cdot}56}{14{\cdot}1246\times10^5}\right] = 3{\cdot}88 \text{ N/mm}^2$$

i.e. $\sigma_C = 3{\cdot}88$ N/mm² **(Ans.)**

(*ii*) Deflection of the beam, δ :

The deflection δ is given by :

$$\delta = \frac{KWl^3}{E}\sqrt{\frac{\sin^2\theta}{I_{VV}^2} + \frac{\cos^2\theta}{I_{UU}^2}} \qquad \text{[Equ. (21·7)]}$$

where, $K = \frac{1}{48}$ for a beam with simply supported ends and carrying a point load at the centre

and, load, $W = 400$ N

length, $l = 2{\cdot}4$ m

Young's modulus, $E = 200\times10^3$ N/mm²

$$I_{UU} = 14{\cdot}1246\times10^5 \text{ mm}^4$$
$$I_{VV} = 3{\cdot}67\times10^5 \text{ mm}^4$$

Substituting the values, we get

$$\delta = \frac{1}{48}\times\frac{400\times(2{\cdot}4\times10^3)^3}{200\times10^3}\sqrt{\frac{(\sin 45°)^2}{(3{\cdot}67\times10^5)^2} + \frac{(\cos 45°)^2}{(14{\cdot}1246\times10^5)^2}}$$

$$= 5.76 \times 10^5 \times \frac{1}{10^5}\sqrt{\frac{1}{2\times(3.67)^2}+\frac{1}{2\times(14.1246)^2}} = 1.1466 \text{ mm}$$

i.e. $\delta = 1.1466$ **mm (Ans.)**

The deflection δ will be inclined at an angle β clockwise with the line GV, given by

$$\tan\beta = \frac{I_{UU}}{I_{VV}}\tan\theta = \frac{14.1246\times10^5}{3.67\times10^5}\tan 45^\circ = 3.848$$

$\therefore$ $\beta = 75.43°$

Thus the deflection is at $75.43° - 45° = 30.43°$ clockwise with the load line GY'.

(iii) **Position of the neutral axis :**

The neutral axis will be at $90° - 30.43° = 59.57°$ *anti-clockwise with the load line*, because the neutral axis is perpendicular to the line of deflection.

Example 21·2. *A cantilever, of I-section, 2·4 metres long is subjected to a load of 200N at the free end as shown in Fig. 21·4. Determine the resulting bending stresses at corners A and B, on the fixed section of the cantilever.*

Solution. Length of the cantilever, $l = 1.8$ m

Load, $W = 600$ N

Refer to Fig. 21·4.

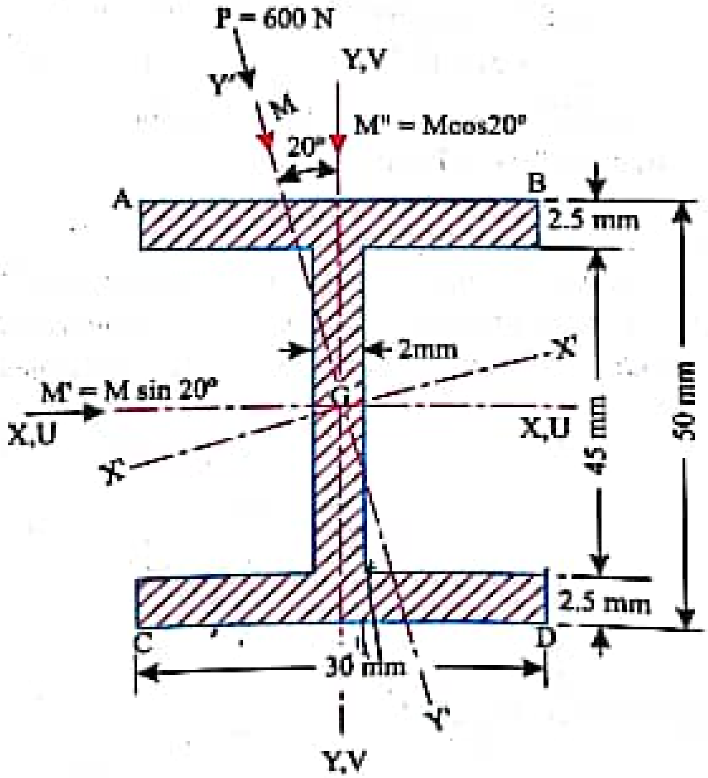


Fig. 21.4

Since I-section is symmetrical about XX and YY axes, therefore XX and YY are the principal axes UU and VV.

$$\text{Moment of inertia, } I_{UU} = I_{XX} = \frac{30\times50^3}{12} - \frac{28\times45^3}{12} = 99875 \text{ mm}^4 = 9.99\times10^{-8} \text{ m}^4$$

$$I_{VV} = I_{YY} = 2\times\frac{2.5\times30^3}{12} + \frac{45\times2^3}{12} = 1.128\times10^{-8} \text{ m}^4$$

Maximum bending moment,

$$M = W \times l = 200 \times 2\cdot 4 = 480 \text{ Nm}$$

Components of M are :

$$M' = M \sin 20° = 480 \times \sin 20° = 164\cdot 17 \text{ Nm}$$

$$M'' = M \cos 20° = 480 \times \cos 20° = 451 \text{ Nm}$$

M' will cause tensile stresses at points A and C and compressive stresses at points B and D

M'' will cause tensile stresses at points A and B and compressive stresses at points C and D

Now, *resultant bending stresses* on A and B are as follows :

$$\sigma_A = \frac{M'' \times (25 \times 10^{-3})}{I_{XX}} + \frac{M' \times (15 \times 10^{-3})}{I_{YY}}$$

$$= \left[\frac{451 \times (25 \times 10^{-3})}{9\cdot 99 \times 10^{-8}} + \frac{164\cdot 17 \times (15 \times 10^{-3})}{11\cdot 28 \times 10^{-8}}\right] \times 10^{-6} \text{ MN/m}^2$$

$$= 112\cdot 86 + 218\cdot 31 = 331\cdot 17 \text{ MN/m}^2$$

i.e. $\quad \boldsymbol{\sigma_A = 331\cdot 17 \text{ MN/mm}^2}$ **(Ans.)**

$$\sigma_B = \frac{M'' \times (2\cdot 5 \times 10^{-3})}{I_{XX}} - \frac{M' \times (1\cdot 5 \times 10^{-3})}{I_{YY}}$$

$$= \left[\frac{451 \times (2\cdot 5 \times 10^{-3})}{9\cdot 99 \times 10^{-8}} - \frac{164\cdot 17 \times (1\cdot 5 \times 10^{-3})}{11\cdot 28 \times 10^{-8}}\right] \times 10^{-6} \text{ MN/m}^2$$

$$= 112\cdot 86 - 218\cdot 31 = -\ 105\cdot 45 \text{ MN/m}^2$$

i.e. $\quad \boldsymbol{\sigma_B = -\ 105\cdot 45 \text{ MN/m}^2}$ **(Ans.)**

Example 21·3. *Fig. 21·5 shows a 80 mm × 80 mm angle having* $I_{XX} = I_{YY} = 87\cdot 36 \times 10^{-8}\ m^4$ *It is used as a freely supported beam with one leg vertical. On the application of the bending moment in the vertical plane YY the mid-section of the beam deflects in the direction AA' at 30° 15' to the vertical.*

(*i*) *Calculate the second moment of area of the section about its principal axis.*

(*ii*) *What is the bending stress at the corner B if the bending moment is 1·5 kNm?*

(Panjab University)

Bridge.

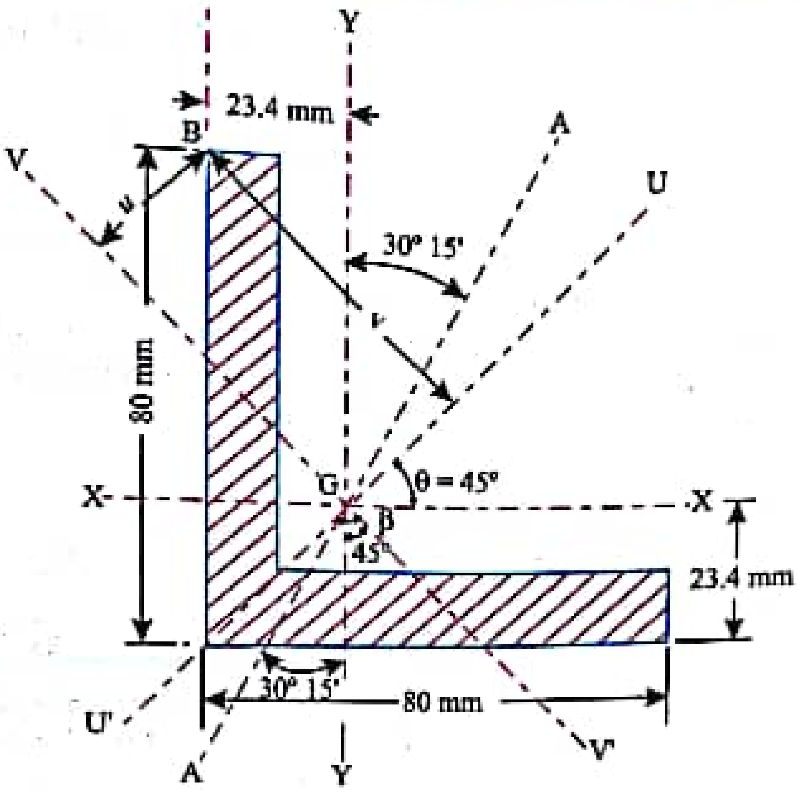


Fig. 21·5

Solution. *Given* : $I_{XX} = I_{YY} = 87{\cdot}36 \times 10^{-8}\ \text{m}^4$

Bending moment, $M = 1{\cdot}5$ kNm

(*i*) **Second moment of area about the principal axes, I_{UU}, I_{YY} :**

We know, $I_{UU} + I_{VV} = I_{XX} + I_{YY}$

$$= 2 \times 87{\cdot}36 \times 10^{-8} = 174{\cdot}72 \times 10^{-8}\ \text{m}^4 \quad ...(i)$$

Also, $\tan \beta = \dfrac{I_{UU}}{I_{VV}} \times \tan \theta$

Here, $\theta = 45°$ (legs of the section being equal)

and, β = Inclination of GA' with $GV' = 45° + 30°\ 15' = 75°\ 15'$

$\therefore$ $\tan (75°\ 15') = \dfrac{I_{UU}}{I_{VV}} \tan 45°$

or $\dfrac{I_{UU}}{I_{VV}} = 3{\cdot}79 \quad ...(ii)$

Solving eqns. (*i*) and (*ii*), we get

$I_{VV} = 36{\cdot}5 \times 10^{-8}\ \text{m}^4$ (Ans.)

and, $I_{UU} = 138{\cdot}2 \times 10^{-8}\ \text{m}^4$ (Ans.)

(*ii*) **Bending stress at B :**

Co-ordinates of B (u, v) :

$$u = 80 \cos 45° - \frac{23{\cdot}4}{\cos 45°} = 23{\cdot}47\ \text{mm} = 0{\cdot}0235\ \text{m}$$

$$v = 80 \sin 45° = 56{\cdot}57\ \text{mm} = 0{\cdot}0566\ \text{m}$$

Now bending stress at B,

$$\sigma_B = M\left[\frac{v \cos \theta}{I_{UU}} + \frac{u \sin \theta}{I_{VV}}\right] \quad \text{Eqn. (21·3)}$$

$$= 1{\cdot}5\left[\frac{0{\cdot}0566\times\cos 45°}{138{\cdot}2\times10^{-8}}+\frac{0{\cdot}0235\times\sin 45°}{36{\cdot}5\times10^{-8}}\right]\times10^{-3}\ \text{MN/m}^2$$

$$= \frac{1{\cdot}5}{10^{-5}}\left[\frac{0{\cdot}0566\times\cos 45°}{138{\cdot}2}+\frac{0{\cdot}0235\times\sin 45°}{36{\cdot}5}\right]$$

$$= 1{\cdot}5\times10^5\,(2{\cdot}895\times10^{-4}+4{\cdot}552\times10^{-4}) = 111{\cdot}7\ \text{MN/m}^2$$

i.e. $\sigma_B = 111{\cdot}7\ \text{MN/m}^2$ **(Ans.)**

Example 21·4. *A beam of T-section (flange : 100 mm × 20 mm ; web : 150 mm × 10 mm) is 2·5 metres in length and is simply supported at the ends. It carries a load of 3·2 kN inclined at 20° to the vertical and passing through the centroid of the section.*

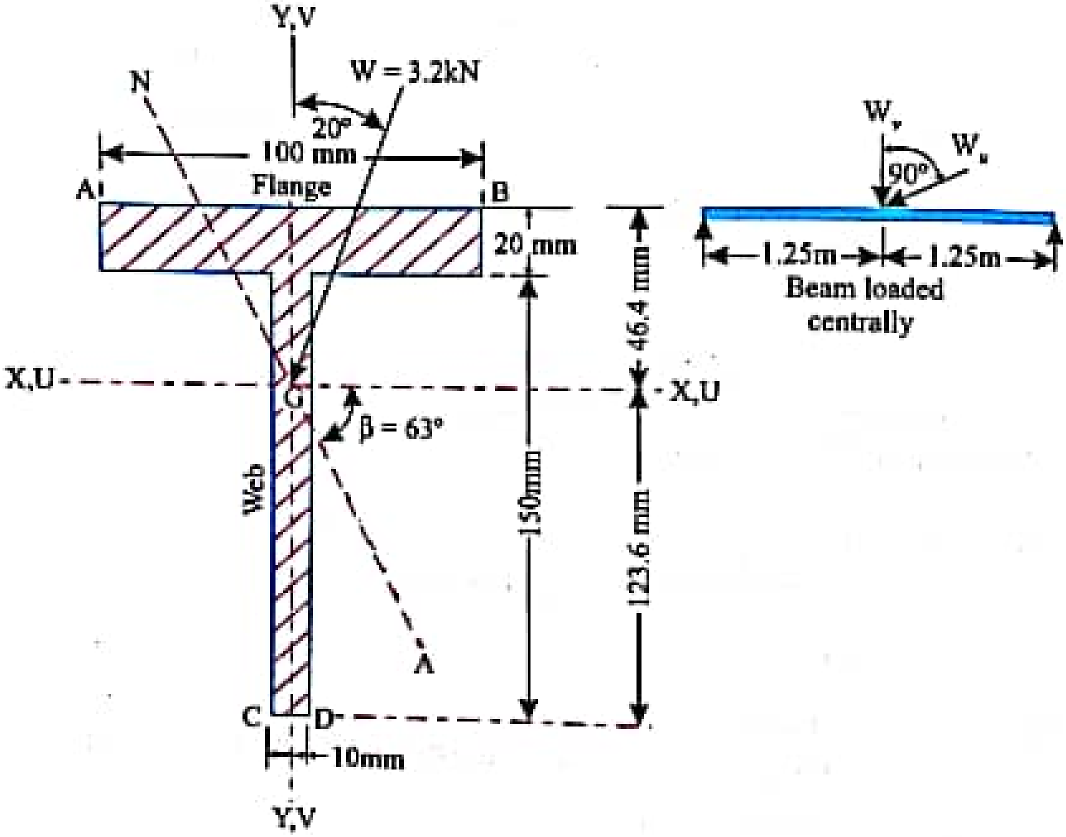


Fig. 21.6

If E = 200 GN/m², calculate :

(i) Maximum tensile stress;

(ii) Maximum compressive stress;

(iii) Deflection due to the load; and

(iv) Position of the neutral axis.

Solution. Length of the beam,

$$l = 2{\cdot}5\ \text{m}$$

Load, $W = 3{\cdot}2\ \text{kN}$

Inclination of the load with the vertical = 20°

Young's modulus, $E = 200\ \text{GN/m}^2$

To find centroid of the T-section taking moments about the top of the flange, we get

$$\bar{y} = \frac{100\times20\times10+150\times10\times\left(\frac{150}{2}+20\right)}{100\times20+150\times10}$$

Scanned by CamScanner

$$= \frac{20000 + 142500}{2000 + 1500} = 46{\cdot}4 \text{ mm}$$

Since the section is symmetrical about the vertical axis, therefore, the principal axes pass ugh the centroid G and are along UU and VV axes shown.

$$I_{XX} = I_{UU} = \left[\frac{100 \times 20^3}{12} + 100 \times 20 \times (46{\cdot}4 - 10)^2\right]$$

$$+ \left[\frac{10 \times 150^3}{12} + 150 \times 10 \times (123{\cdot}6 - 75)^2\right]$$

$$= [66666{\cdot}67 + 2649920] + [2812500 + 3542940]$$

$$= 9{\cdot}07 \times 10^6 \text{ mm}^4 = 9{\cdot}07 \times 10^{-6} \text{ m}^4$$

$$I_{YY} = I_{VV} = \left[\frac{12 \times 100^3}{12} + \frac{150 \times 10^3}{12}\right]$$

$$= 1{\cdot}679 \times 10^6 \text{ mm}^4 = 1{\cdot}679 \times 10^{-6} \text{ m}^4$$

Components of W :

$$W_u = W \sin 20° = 3{\cdot}2 \times \sin 20° = 1{\cdot}094 \text{ kN}$$

$$W_v = W \cos 20° = 3{\cdot}2 \times \cos 20° = 3{\cdot}007 \text{ kN}$$

Bending moments :

$$M_u = \frac{W_u \times l}{4} = \frac{1{\cdot}094 \times 2{\cdot}5}{4} = 0{\cdot}684 \text{ kNm}$$

$$M_v = \frac{W_v \times l}{4} = \frac{3{\cdot}007 \times 2{\cdot}5}{4} = 1{\cdot}879 \text{ kNm}$$

M_u will cause maximum compressive stresses at B and D and maximum tensile stresses at A and C.
M_v will cause maximum compressive stresses at A and B and maximum tensile stresses at C and D.

(i) Maximum tensile stress :

Maximum tensile stress at C,

$$\sigma_c = \frac{M_u \times (5 \times 10^{-3})}{I_{VV}} + \frac{M_v \times (123{\cdot}6 \times 10^{-3})}{I_{UU}}$$

$$= \left[\frac{0{\cdot}684 \times 5 \times 10^{-3}}{1{\cdot}679 \times 10^{-6}} \times 10^{-3} + \frac{1{\cdot}879 \times 123{\cdot}6 \times 10^{-3}}{9{\cdot}07 \times 10^{-6}} \times 10^{-3}\right] \text{MN/m}^2$$

$$= 2{\cdot}04 + 25{\cdot}6 = 27{\cdot}64 \text{ MN/m}^2$$

i.e. $\sigma_C = 27{\cdot}64$ MN/m² **(Ans.)**

(ii) Maximum compressive stress :

Maximum compressive stress at B,

$$\sigma_B = \frac{M_u \times (50 \times 10^{-3})}{I_{VV}} + \frac{M_v \times (46{\cdot}4 \times 10^{-3})}{I_{UU}}$$

$$= \left[\frac{0{\cdot}684 \times (50 \times 10^{-3})}{1{\cdot}679 \times 10^{-6}} \times 10^{-3} + \frac{1{\cdot}879 \times 46{\cdot}4 \times 10^{-3}}{9{\cdot}07 \times 10^{-6}} \times 10^{-3}\right] \text{MN/m}^2$$

$$= 20{\cdot}37 + 9{\cdot}61 = 29{\cdot}98 \text{ MN/m}^2$$

i.e. $\sigma_B = 29{\cdot}98$ MN/m² **(Ans.)**

(iii) Deflection due to the load, δ :

We know that;

$$\delta = \frac{KWl^3}{E}\sqrt{\frac{\sin^2\theta}{I_{VV}^2}+\frac{\cos^2\theta}{I_{UU}^2}}$$

where, $K = \frac{1}{48}$ for a beam with simply supported ends, and carrying a point load at its centre,

or, $$\delta = \frac{KWl^3}{EI_{UU}}\sqrt{\sin^2\theta\times\left(\frac{I_{UU}}{I_{VV}}\right)^2+\cos^2\theta}$$

$$= \frac{1}{48}\times\frac{3.2\times10^3\times(2.5)^3}{200\times10^9\times9.07\times10^{-6}}\sqrt{(\sin 20°)^2\times\left(\frac{9.07\times10^{-6}}{1.679\times10^{-6}}\right)+(\cos 20°)^2}$$

$$= 5.742\times10^{-4}\sqrt{3.414+0.883} = 11.9\times10^{-4}\text{ m} = 1.19\text{ mm}$$

i.e. $\delta = 1.19$ **mm (Ans.)**

(iv) Position of the neutral axis :

We know that,

$$\tan\beta = \frac{I_{UU}}{I_{VV}}\tan\theta = \frac{9.07\times10^{-6}}{1.679\times10^{-6}}\times\tan 20° = 1.966 \quad \text{[Eqn. 21.8]}$$

∴ $\beta = 63°$ **(Ans.)**

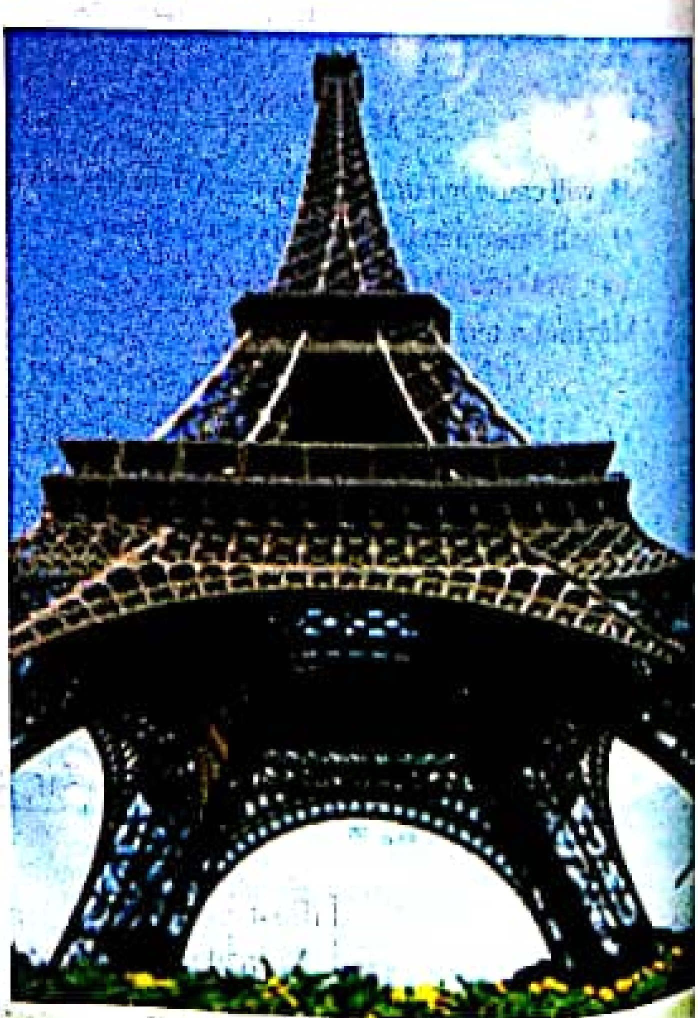
21·5. SHEAR CENTRE

— The shear centre (*for any transverse section of the beam*) *is the point of intersection of the bending axis and the plane of the transverse section.*

— Shear centre of a section can be defined *as a point about which the applied force is balanced by the set of shear forces obtained by summing the shear stresses over the section* (for unsymmetrical sections such as angle section and channel section, summation of shear stresses in each leg gives a set of forces which should be in equilibrium with the applied shear force).

Shear centre is also known as "*centre of twist.*"

— In case of a beam having *two axes of symmetry, the shear centre coincides with the centroid.*



Eiffel Tower, Paris.

– In case of sections having one axis of symmetry, the shear centre *does not coincide with the centroid but lies on the axis of symmetry.*

– *When the load passes through the shear centre then there will be only bending in the cross-section and no twisting.*

The principle involved in locating the shear centre for a cross-section of a beam is that the loads acting on the beam must lie in a plane which contains the resultant shear force on each cross-section of the beam as computed from the shearing stresses produced in the beam when it is loaded so that it does not twist at its ends.

21·5·1. Shear Centre for Channel Section

Fig. 21·7 shows a channel section (flanges : $b \times t_1$; *web* : $h \times t_2$) with XX as the horizontal symmetric axis.

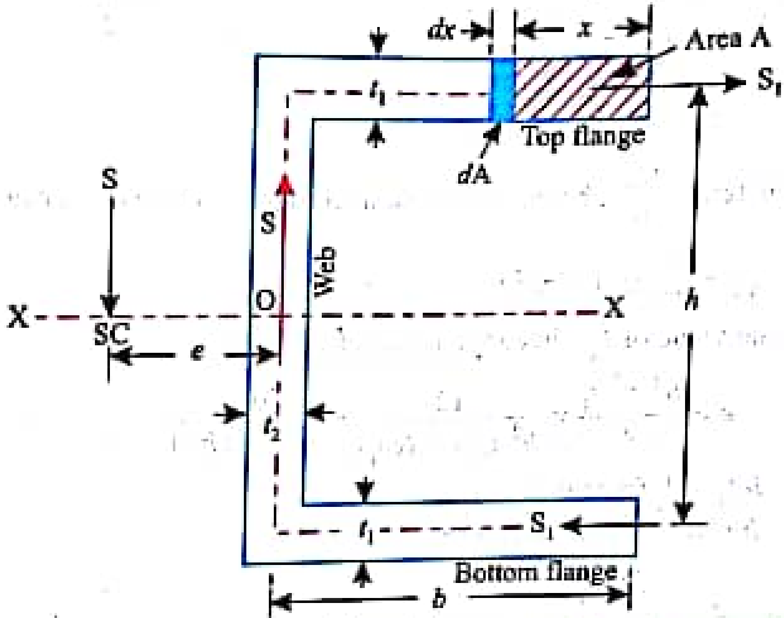


Fig. 21.7

Let, S = Applied shear force (vertical downwards), and

(then S is the shear force in the web in the upward direction).

S_1 = Shear force in the top flange (there will be *equal and opposite shear force in the bottom flange as shown).*

Now, shear stress (τ) in the flange at a distance of x from the right hand edge (of the top flange),

$$\tau = \frac{SA\bar{y}}{I_{XX}t}$$

$$A\bar{y} = (t_1 \cdot x)\frac{h}{2} \qquad \text{(where, } t = t_1\text{, thickness of the flange)}$$

$$\therefore \quad \tau = \frac{St_1 \cdot x}{I_{XX} \cdot t_1} \cdot \frac{h}{2} = \frac{S\,x\,h}{2I_{XX}}$$

Shear force in elementary area

$$(dA = t_1 \cdot dx) = \tau \cdot dA = \tau \cdot t_1 \cdot dx$$

Total shear force in top flange

$$= \int_0^b \tau \cdot t_1 \cdot dx \qquad \text{(where, } b = \text{breadth of the flange)}$$

$$S_1 = \int_0^b \frac{S\,x\,h}{2I_{XX}} \cdot t_1 \cdot dx = \frac{Sht_1}{2I_{XX}}\int_0^b x\,dx$$

or, $$S_1 = \frac{Sh\,t_1}{I_{XX}} \cdot \frac{b^2}{4}$$

Let, e = Distance of the shear centre (SC) from the web along the symmetric axis XX.

Taking moments of shear forces about the centre O of the web, we get

$$S \cdot e = S_1 \cdot h$$

$$= \frac{Sh\,t_1}{I_{XX}} \cdot \frac{b^2}{4} \cdot h = \frac{S \cdot t_1\, h^2\, b^2}{4\, I_{XX}}$$

$\therefore$ $$e = \frac{b^2\, h^2\, t_1}{4\, I_{XX}} \quad \ldots(i)$$

Now, $$I_{XX} = 2\left[\frac{b \times t_1^3}{12} + b \cdot t_1 \left(\frac{h}{2}\right)^2\right] + \frac{t_2\, h^3}{12} = \frac{b\, t_1^3}{6} + \frac{b\, t_1\, h^2}{2} + \frac{t_2\, h^3}{12}$$

$$= \frac{b\, t_1\, h^2}{2} + \frac{t_2\, h^3}{12}$$

(neglecting the term $\frac{b\, t_1^3}{6}$, being negligible in comparison to other terms)

or, $$I_{XX} = \frac{h^2}{12}(t_2 h + 6bt_1)$$

Substituting the value of I_{XX} in eqn. (i), we get

$$e = \frac{b^2\, h^2\, t_1}{4} \times \frac{12}{h^2\,(t_2 h + 6bt_1)} = \frac{3b^2\, t_1}{(t_2 h + 6bt_1)}$$

Let, $b\, t_1 = A_f$ (area of the flange)

and, $h\, t_2 = A_w$ (area of the web)

Then, $$e = \frac{3b\, A_f}{A_w + 6A_f} = \frac{3b}{6 + \frac{A_w}{A_f}}$$

i.e. $$e = \frac{3b}{6 + \frac{A_w}{A_f}} \quad \ldots(21\cdot9)$$

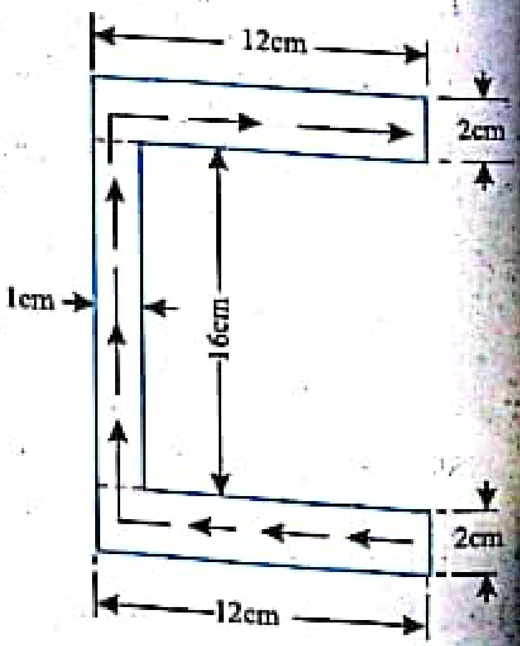


Fig. 21.8

Example 21·5. *A channel section has flanges 12 cm × 2 cm and web 16 cm × 1 cm. Determine the shear centre of the channel.*

Solution. Here, $b = 12 - 0{\cdot}5 = 11{\cdot}5$ cm

$t_1 = 2$ cm, $t_2 = 1$ cm, $h = 18$ cm

$A_w = h\, t_2 = 18 \times 1 = 18$ cm^2

and, $A_f = bt_1 = 11{\cdot}5 \times 2 = 23$ cm^2

We know that, $$e = \frac{3b}{6 + \frac{A_w}{A_f}} \quad \text{(Eqn. 21·9)}$$

$$= \frac{3 \times 11{\cdot}5}{6 + \frac{18}{23}} = 5{\cdot}086 \text{ cm}$$

Hence, position of the shear centre = **5·086 cm** **(Ans.)**

52. Shear Centre for Unequal I-section

Fig. 21·9 shows an unequal I-section which is symmetrical about XX axis.

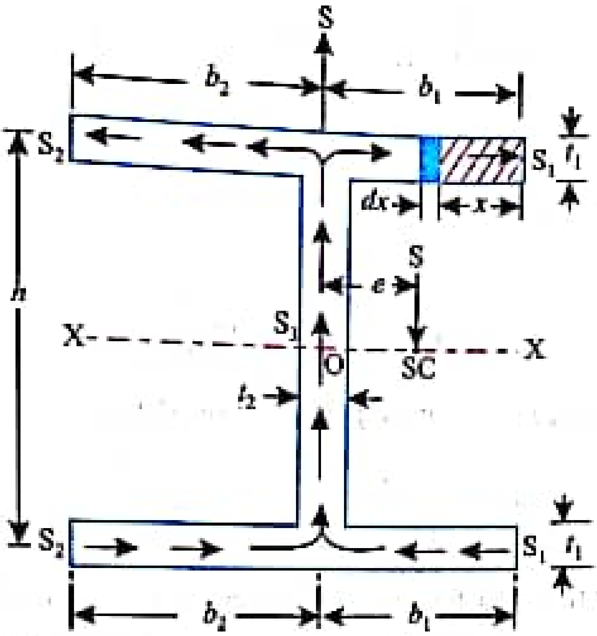


Fig. 21.9

Shear stress in any layer, $\tau = \dfrac{SA\,\bar{y}}{It}$

where, $$I = I_{XX} = 2\left[(b_1 + b_2)\frac{t_1^3}{12} + (b_1 + b_2)\, t_1 \times \frac{h^2}{4}\right] + t_2 \cdot \frac{h^3}{12}$$

...section of the earthmover's bucket is not symmetical althrough. But due to the nature of work this asymmetry to some extent, is unavoidable.

Shear force S_1 :

$$dA = t_1 \cdot dx,\ A\bar{y} = t_1 \cdot x \cdot \frac{h}{2}$$

$$S_1 = \int_0^{b_1} \tau\, dA = \frac{S \cdot x \cdot t_1}{I_{XX} \cdot t_1} \cdot \frac{h}{2} \times t_1\ dx$$

$$= \int_0^{b_1} \frac{S}{2I_{XX}} \times h \cdot t_1 \cdot x\, dx$$

$$= \frac{Sht_1}{2I_{XX}} \left| \frac{x^2}{2} \right|_0^{b_1} = \frac{Sh\, t_1\, b_1^2}{4I_{XX}}$$

Similarly the shear force (S_2) in the other part of the flange,

$$S_2 = \frac{Sh\, t_1\, b_2^2}{4I_{XX}}$$

Taking moments of the shear forces about the centre of the web O, we get

$$S_2 \cdot h = S_1 \cdot h + S \cdot e \qquad (S_3 = S \text{ for equilibrium})$$

(where, e = distance of shear centre from the centre of the web)

or, $(S_2 - S_1)\, h = S \cdot e$

$$\frac{Sh^2\, t_1}{4I_{XX}} (b_2^2 - b_1^2) = S \cdot e$$

$$\therefore \qquad e = \frac{t_1\, h^2\, (b_2^2 - b_1^2)}{4I_{XX}} \qquad ...(21\cdot$$

Example 21·6. *Determine the position of the shear centre of the section of a beam shown Fig. 21·10.*

Solution. Here, $t_1 = 4$ cm.

$b_1 = 6$ cm

$b_2 = 8$ cm

$h = 30 - 4 = 26$ cm

$$I_{XX} = 2\left[\frac{14 \times 4^3}{12} + 14 \times 4 \times 13^2\right] + \frac{2 \times 22^3}{12}$$

$$= 2\,(74{\cdot}67 + 9464) + 1774{\cdot}67 = 20852 \text{ cm}^4$$

We know, $$e = \frac{t_1\, h^2\, (b_2^2 - b_1^2)}{4I_{XX}}$$

(where, e = distance of the shear centre from the centre of the web)

$$\therefore \qquad e = \frac{4 \times 26^2\, (8^2 - 6^2)}{4 \times 20852} = 0{\cdot}9077 \text{ cm} \quad \textbf{(Ans.)}$$

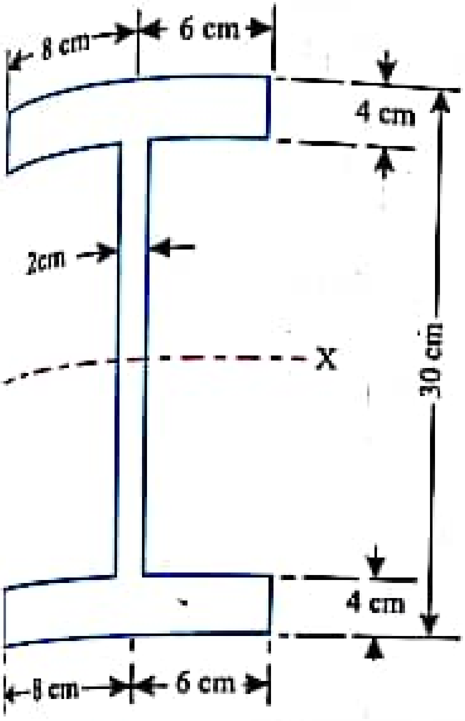


Fig. 21·10

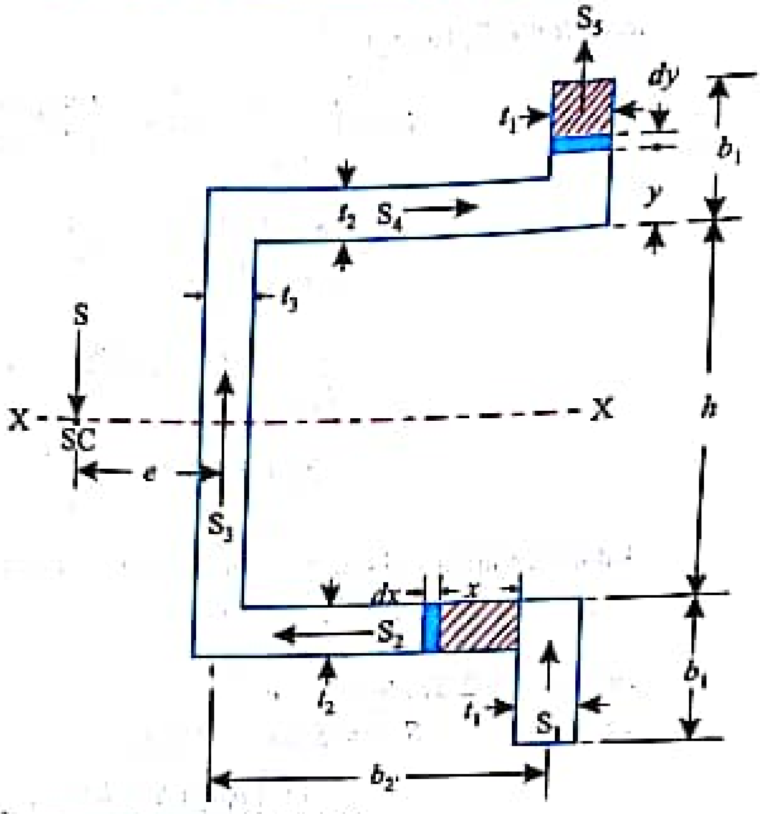


Fig. 21·11

Example 21·7. *Locate the shear centre of the section shown in the Fig. 21·11*

Solution. Refer to Fig. 21·11.

Since the section is symmetrical about *XX* axis the shear centre will lie on this axis.

Also by symmetry: Shear forces $S_1 = S_5$, $S_2 = S_4$

Shear force S_1 (or S_5).

$$\tau = \frac{SA\,\bar{y}}{I_{XX} \cdot t_1} = \frac{S\,(b_1 - y)\,t_1}{I_{XX} \cdot t_1} \times \left[\frac{h}{2} + y + \frac{b_1 - y}{2}\right]$$

$$= \frac{S\,(b_1 - y)}{I_{XX}}\left[\frac{h + b_1 + y}{2}\right]$$

(where, S = applied shear force on the section)

Now, $dA = t_1 \cdot dy$

Shear force

$$S_1 = \int_0^{b_1} \frac{S\,(b_1 - y)}{2I_{XX}}\,(h + b_1 + y)\,t_1\,dy$$

$$= \frac{S\,t_1}{2I_{XX}} \int_0^{b_1} (hb_1 - hy + b_1^2 - b_1 y + b_1 y - y^2)\,dy$$

$$= \frac{S\,t_1}{2I_{XX}} \left|(hb_1 y - \frac{hy^2}{2} + b_1^2 y - \frac{y^3}{3})\right|_0^{b_1}$$

$$= \frac{S\,t_1}{2I_{XX}}\left[b_1^2 h - \frac{h}{2}\,b_1^2 + b_1^3 - \frac{b_1^3}{3}\right]$$

$$= \frac{S\,t_1}{2I_{XX}}\left[\frac{b_1^2 h}{2} + \frac{2b_1^3}{3}\right] = \frac{S b_1^2\,t_1}{12\,I_{XX}}\,(3h + 4b_1)$$

Shear force S_2 (or S_4).

$$S_2 = \int_0^{b_2} \frac{S}{I_{XX} \cdot t_2} \times \left[b_1 t_1 \left(\frac{h}{2} + \frac{b_1}{2} \right) + t_2 x \cdot \frac{h}{2} \right] t_2 \, dx$$

$$= \frac{S}{I_{XX}} \int_0^{b_2} \left(\frac{b_1 t_1 h}{2} + \frac{b_1^2 t_1}{2} + \frac{t_2 h}{2} \cdot x \right) dx$$

$$= \frac{S}{I_{XX}} \left| \frac{b_1 t_1 h}{2} \cdot x + \frac{b_1^2 t_1}{2} \cdot x + \frac{t_2 h}{2} \cdot \frac{x^2}{2} \right|_0^{b_2}$$

$$= \frac{S}{I_{XX}} \left[\frac{b_1 b_2 t_1 h}{2} + \frac{b_2 b_1^2 t_1}{2} + \frac{t_2 b_2^2 h}{4} \right]$$

Taking moments about the centre of web, we get

$$S \times e + 2 S_1 \cdot b_2 = 2 S_2 \times \frac{h}{2}$$

or, $S \cdot e + 2 S_1 \cdot b_2 = S_2 \cdot h$

or, $S \cdot e = S_2 \cdot h - 2 S_1 \cdot b_2$

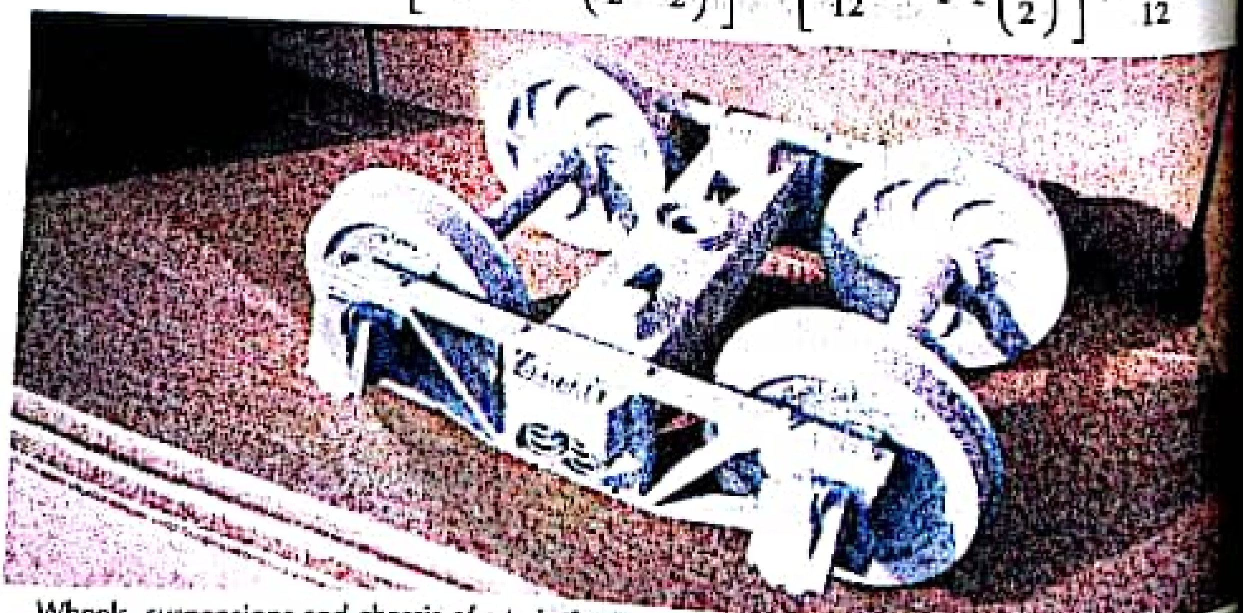
$$= \frac{Sh}{I_{XX}} \left[\frac{b_1 b_2 t_1 h}{2} + \frac{b_1^2 b_2 t_1}{2} + \frac{b_2^2 t_2 h}{4} \right] - \frac{S b_1^2 b_2 t_1}{6 I_{XX}} (3h + 4b_1)$$

$$\therefore \quad e = \frac{h^2}{I_{XX}} \left(\frac{b_1 b_2 t_1}{2} + \frac{t_2 b_2^2}{4} \right) + \frac{h\, b_1^2 b_2 t_1}{2 I_{XX}} - \frac{b_1^2 b_2 t_1 h}{2 I_{XX}} - \frac{2}{3} \frac{b_1^3 b_2 t_1}{I_{XX}}$$

$$= \frac{h^2}{I_{XX}} \times \frac{b_1 b_2 t_1}{2} + \frac{h^2 t_2 b_2^2}{4 I_{XX}} - \frac{2}{3} \frac{b_1^3 b_2 t_1}{I_{XX}}$$

or, $$e = \frac{b_1 b_2 t_1}{I_{XX}} + \left[\frac{h^2}{2} - \frac{2}{3} b_1^2 \right] + \frac{h^2 b_2^2 t_2}{4 I_{XX}}$$

where, $$I_{XX} = 2 \left[\frac{t_1 b_1^3}{12} + b_1 t_1 \left(\frac{b_1}{2} + \frac{h}{2} \right)^2 \right] + 2 \left[\frac{b_2 t_2^3}{12} + b_2 t_2 \left(\frac{h}{2} \right)^2 \right] + \frac{t_3 h^3}{12}$$



Wheels, suspensions and chassis of a train freight car. The centre transverse member has U-cross-section.

Example 21·8. *Find the shear centre of the section shown in Fig. 21·12.*

Solution. Refer to Fig. 21·12

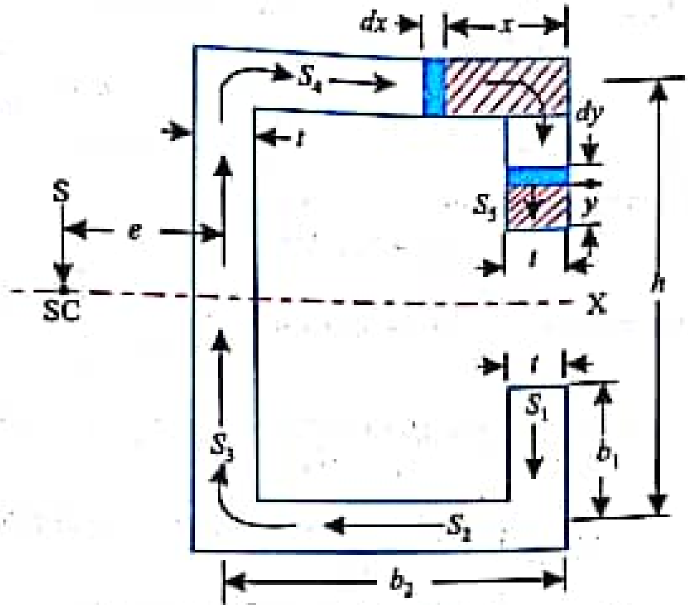


Fig. 21.12

Flanges : $b_2 \times t$

Web : $h \times t$

Projections : $b_1 \times t$

Let, S = Applied force, and

S_1, S_2, S_3, S_4, S_5 = Shear forces in different portions.

Also, $S_1 = S_5$, and $S_2 = S_4$ (By symmetry)

Shear stress in any layer, $\tau = \dfrac{SA\bar{y}}{I_{xx} \cdot t}$

Shear force S_1 ($= S_5$).... vertical projection

Area, $A = y \cdot t$

$$\bar{y} = \left(\frac{h}{2} - b_1\right) + \frac{y}{2} = \left(\frac{h - 2b_1 + y}{2}\right)$$

Area, $dA = t \cdot dy$

∴ Shear force
$$S_5 = \int_0^{b_1} \tau \cdot dA = \int_0^{b_1} \frac{Syt}{I_{xx} \cdot t} \times \left(\frac{h - 2b_1 + y}{2}\right) t \cdot dy$$

$$= \frac{St}{2I_{xx}} \int_0^{b_1} (hy - 2b_1 y + y^2)\, dy = \frac{St}{2I_{xx}} \left| \frac{hy^2}{2} - b_1 y^2 + \frac{y^3}{3} \right|_0^{b_1}$$

$$= \frac{St}{2I_{xx}} \left[\frac{hb_1^2}{2} - b_1^3 + \frac{b_1^3}{3}\right] = \frac{St}{2I_{xx}} \left[\frac{hb_1^2}{2} - \frac{2}{3} b_1^3\right]$$

or,
$$S_5 = \frac{St\, b_1^2}{12\, I_{xx}} (3h - 4b_1)$$

Shear force, S_4 ($= S_2$) ...Flange

$$A\bar{y} = (x \cdot t)\,\frac{h}{2} + b_1 \cdot t \left(\frac{h}{2} - b_1 + \frac{b_1}{2}\right) = x \cdot t \cdot \frac{h}{2} + b_1 \cdot t \left(\frac{h}{2} - \frac{b_1}{2}\right)$$

and, $dA = dx \cdot t$

$$\therefore \quad S_4 = \int_0^{b_2} \frac{S}{I_{XX} \cdot t} \times \left[\frac{x.t.h}{2} + \frac{b_1 t}{2}(h - b_1)\right] \times dx.t$$

$$= \frac{St}{2I_{XX}} \int_0^{b_2} (x.h + b_1 h - b_1^2)\, dx = \frac{St}{2I_{XX}} \left[\frac{b_2^2 h}{2} + b_1 b_2 h - b_1^2 b_2\right]$$

Taking moments about the centre of the web, we get

$$S.e = 2 \times S_5 \times b_2 + 2 \times S_4 \times h/2$$

or, $$S.e = 2 \times \frac{St\, b_1^2}{12\, I_{XX}}(3h - 4b_1) \times b_2 + 2 \times \frac{St}{2I_{XX}}\left[\frac{b_2^2 h}{2} + b_1 b_2 h - b_1^2 b_2\right] \times \frac{h}{2}$$

or, $$e = \frac{t\, b_1^2 b_2}{6\, I_{XX}}(3h - 4b_1) + \frac{t.h}{4I_{XX}}(b_2^2 h + 2b_1 b_2 h - 2b_1^2 b_2)$$

$$= \frac{1}{12\, I_{XX}}\left[6t\, b_1^2 b_2 h - 8t\, b_1^3 b_2 + 3t\, h^2 b_2^2 + 6t\, b_1 b_2 h^2 - 6t\, h b_1^2 b_2\right]$$

$$= \frac{t}{12\, I_{XX}}\left[6b_1^2 b_2 h - 8b_1^3 b_2 + 3h^2 b_2^2 + 6b_1 b_2 h^2 - 6h b_1^2 b_2\right]$$

or, $$e = \frac{t}{12\, I_{XX}}\left[-8b_1^3 b_2 + 3h^2 b_2^2 + 6b_1 b_2 h^2\right]$$

where, $$I_{XX} = \frac{t \times h^3}{12} + 2\left[\frac{b_2 \times t^3}{12} + b_2 t \times \left(\frac{h}{2}\right)^2\right] + \left[\frac{t \times b_1^3}{12} + b_1 t \times \left(\frac{h}{2} - \frac{b_1}{2}\right)^2\right]$$

or, $$I_{XX} = \frac{t.h^3}{12} + \frac{b_2.t^3}{6} + \frac{b_2.t.h^2}{2} + \frac{t.b_1^3}{6} + \frac{b_1.t}{2}(h - b_1)^2$$

Example 21·9. *Locate the shear centre of the section shown in Fig. 21·13.*

Solution. Refer to Fig. 21·13.

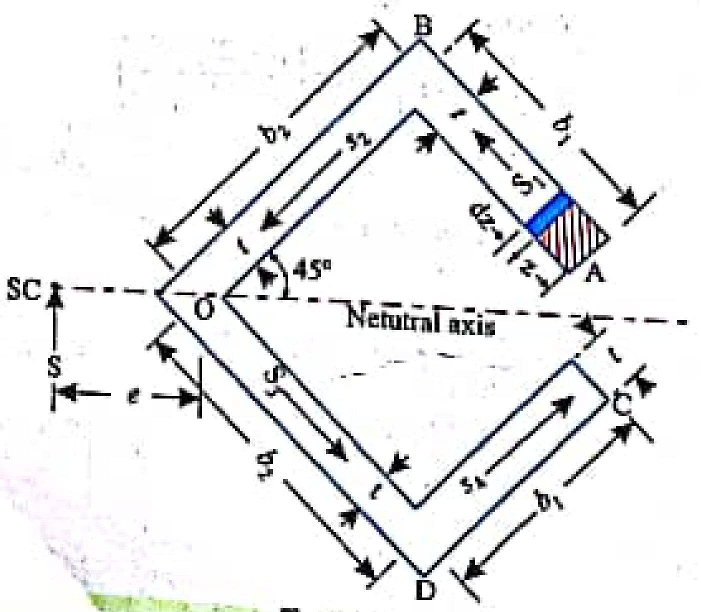


Fig. 21.13

$S_1 = S_4$, and, $S_2 = S_3$ ... *Due to symmetry*

Shear stress in any layer, $\tau = \dfrac{SA\bar{y}}{I_{NA}}$

where, S = Applied force, and I_{NA} = Moment of inertia about neutral axis.

Shear force S_1 $(= S_4)$.

$$S_1 = \int_0^{b_1} \tau . dA$$

where, $A = z . t$

$dA = t\, dz$

$\therefore$

$$\bar{y} = (b_2 \sin 45° - b_1 \sin 45°) + \frac{z}{2} \sin 45°$$

$$= \left(b_2 - b_1 + \frac{z}{2}\right) \sin 45° = \frac{2b_2 - 2b_1 + z}{2\sqrt{2}}$$

$$\therefore \quad S_1 = \int_0^{b_1} \frac{Sz.t}{I_{NA}.t} \left(\frac{2b_2 - 2b_1 + z}{2\sqrt{2}}\right) t.dz = \frac{St}{2\sqrt{2}\, I_{NA}} \int_0^{b_1} (2b_2\, z - 2b_1\, z + z^2)\, dz$$

$$= \frac{St}{2\sqrt{2}\, I_{NA}} \left[b_2 b_1^2 - b_1^3 + \frac{b_1^3}{3}\right] = \frac{St\, b_1^2\, (3b_2 - 2b_1)}{6\sqrt{2}\, I_{NA}}$$

Total moment of inertia of the section :

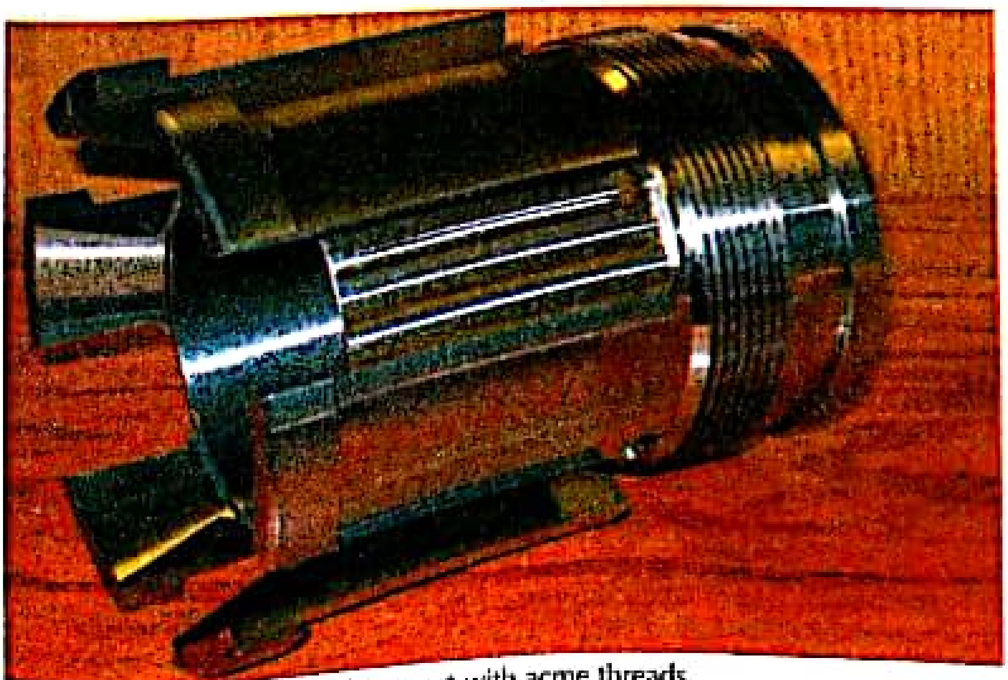
For rectangle I :

$$I_{U_1U_1} = \frac{b_1 t^3}{12},\ I_{V_1V_1} = \frac{t.b_1^3}{12}$$

Now, $I_{X_1X_1} = I_{U_1U_1} \cos^2\theta + I_{V_1V_1} \sin^2\theta = \dfrac{I_{UU} + I_{VV}}{2}$ $\qquad (\because \theta = 45°)$

or, $$I_{X_1X_1} = \frac{1}{2}\left(\frac{b_1 t^3}{12} + \frac{t b_1^3}{12}\right) = \frac{b_1 t}{24}(b_1^2 + t^2)$$

$$I_{NA_1} = I_{X_1X_1} + t.b_1 \left[\left(b_2 - \frac{b_1}{2}\right)\sin 45°\right]^2 = I_{X_1X_1} + \frac{t b_1\, (2b_2 - b_1)^2}{8}$$



Machine part with acme threads.

$$= \frac{b_1 t}{24}(b_1^2 + t^2) + b_1 t\left[\frac{b_2^2}{2} + \frac{b_1^2}{8} - \frac{b_1 b_2}{2}\right] = \frac{b_1 t}{24}[b_1^2 + t^2 + 12b_2^2 + 3b_1^2 - 12b_1 b_2]$$

or, $$I_{NA_1} = \frac{b_1 t}{24}[t^2 + 4b_1^2 + 12b_2^2 - 12b_1 b_2]$$

For rectangle II :

$$I_{U_2U_2} = \frac{b_2 t^3}{12}, \quad I_{V_2V_2} = \frac{t b_2^3}{12}$$

Now, $$I_{X_2X_2} = I_{U_2U_2}\cos^2\theta + I_{V_2V_2}\sin^2\theta = \frac{I_{U_2U_2} + I_{V_2V_2}}{2} \qquad (\because \theta = 45°)$$

$$= \frac{1}{2}\left(\frac{b_2 t^3}{12} + \frac{t b_2^3}{12}\right) = \frac{b_2 t}{24}(b_2^2 + t^2)$$

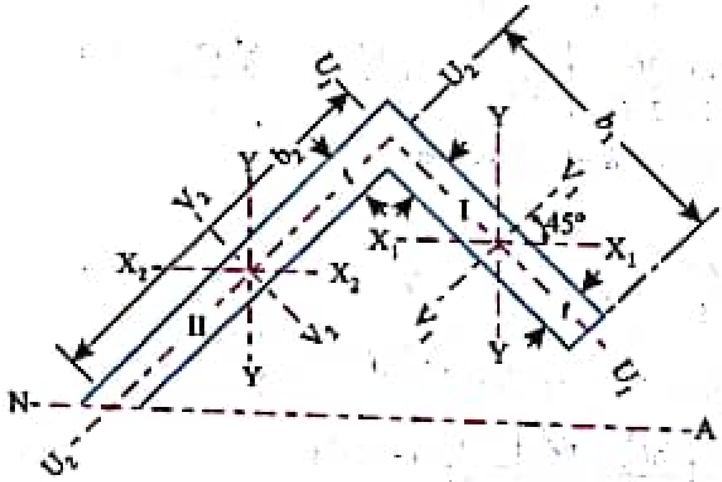


Fig. 21.14

$$I_{NA_2} = I_{X_2X_2} + b_2 . t\left(\frac{b^2}{2}\sin 45°\right)^2 = \frac{b_2 t}{24}(b_2^2 + t^2) + b_2 t\left(\frac{b_2^2}{8}\right)$$

$$= \frac{b_2 t}{24}(4b_2^2 + t^2)$$

Total moment of inertia of the section,

$$I_{NA} = 2 \times I_{NA_1} + 2I_{NA_2}$$

or, $$I_{NA} = \frac{b_1 t}{12}(t^2 + 4b_1^2 + 12b_2^2 - 12b_1 b_2) + \frac{b_2 t}{12}(4b_2^2 + t^2)$$

Taking moments of the shear forces about O, we get

$$S \times e = S_1 \times b_2 + S_1 \times b_2 = 2\,S_1\,b_2$$

$$= \frac{St\,b_1^2 b_2}{3\sqrt{2}\,I_{NA}}(3b_2 - 2b_1)$$

$$\therefore \qquad e = \frac{t\,b_1^2 b_2}{3\sqrt{2}\,I_{NA}}(3b_2 - 2b_1)$$